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Beyond Symmetry: Topological Lines in 2D

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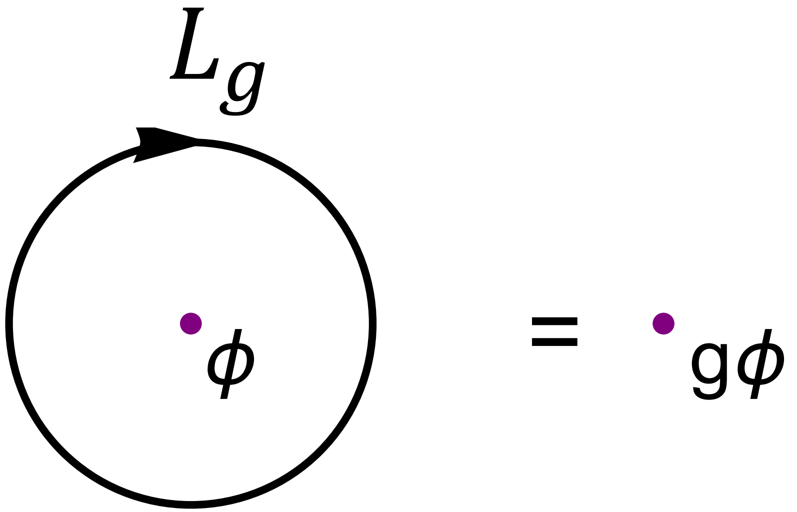
The Power of Symmetry

- Global symmetries are powerful in constraining RG flows, correlation functions, and other physical observables in QFT.
  - ’t Hooft anomaly matching, Ward identity of correlators, checking dualities…
- More recently, higher form global symmetries have also shown to be useful in constraining the dynamics of QFT [Kapustin-Seiberg, Gaiotto-Kapustin-Seiberg-Willet…].
- Global symmetries can be defined in terms of their charge operators, i.e. topological defects.

Symmetry and Topological Defect

- Continuous global symmetry → Noether charge operator
- More generally, a 0-form global symmetry $g \in G$ (continuous or discrete) is associated to a codimension-1 topological defect L_g.
- Topological defect acts on local operators by symmetry transformation.



$g \in G$

0-form Codimension-1

Global Symmetry $\longrightarrow$ Topological Defect

$\overset{?}{\longleftarrow}$

NO

The Power of Non-Symmetry

- There are codimension-1 topological defects that are **NOT** the charge operators of any 0-form global symmetries.
- These non-symmetry topological defects are equally powerful in constraining the dynamics of QFT.
- We don’t have to look far for such defects – they are ubiquitous in 2D CFT (e.g. 2D Ising model).

Roadmap

q-form Global Symmetry
& Symmetry Codim-(q+1) Defect

$\uparrow$

0-form Global Symmetry $\longrightarrow$ Non-Symmetry Codim-1 Defect
& Symmetry Codim-1 Defect

[Bhardwaj-Tachikawa]

Today: (Non-)Symmetry Topological Lines in 2D CFT

Theorem

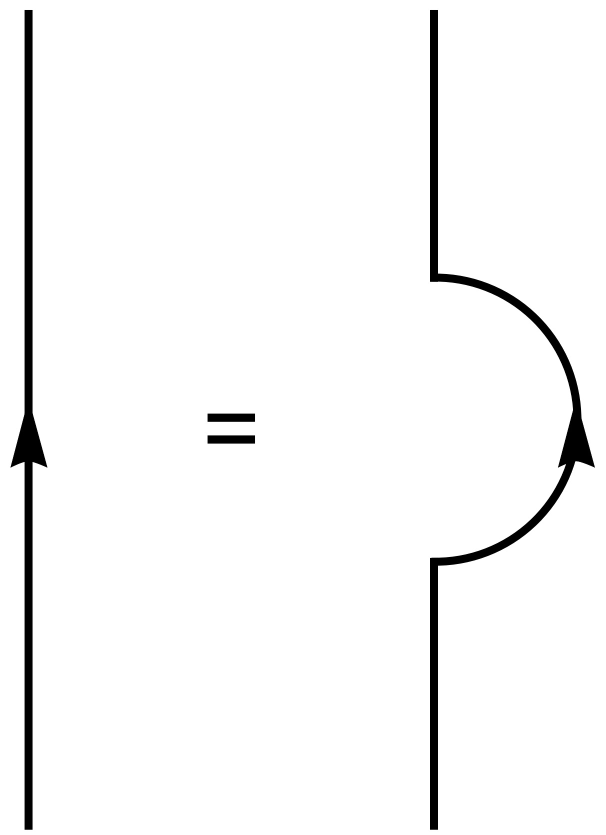
- In 2D, if certain non-symmetry topological line is preserved along a RG flow, then the IR theory can NOT be trivially gapped [Chang-Lin-SHS-Wang-Yin].
    - Similar in spirit to ‘t Hooft anomaly matching

Basic Properties of Topological Lines

1. They are topological

- All physical observables are invariant under continuous deformation of topological lines.

- They commute with **both** Virasoro algebras.



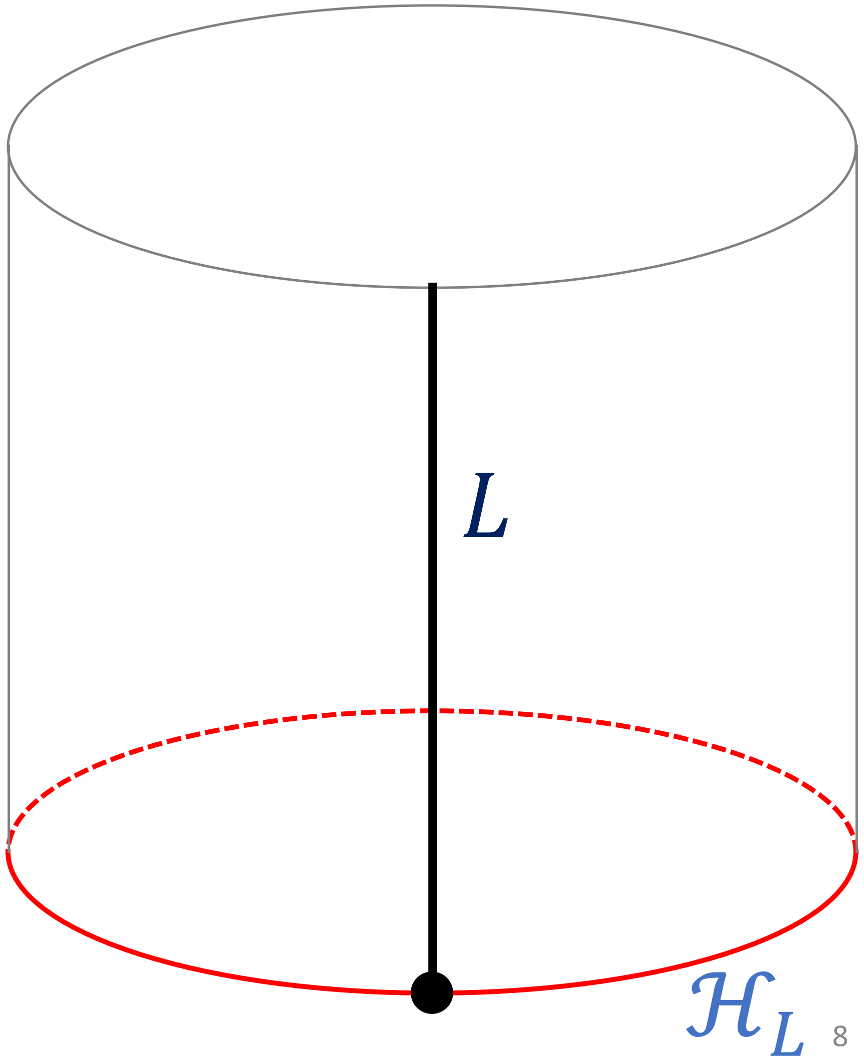
Basic Properties of Topological Lines

2. Defect Hilbert Space $\mathcal{H}_L$

- Followed from the topological property, states in $\mathcal{H}_L$ are in representations of **both** Virasoro algebras.

$$\mathcal{H}_L = \sum_{h,\bar{h}} n_{h,\bar{h}}\, Vir_h \otimes \overline{Vir}_{\bar{h}}$$

$$n_{h,\bar{h}} \in \mathbb{N}$$

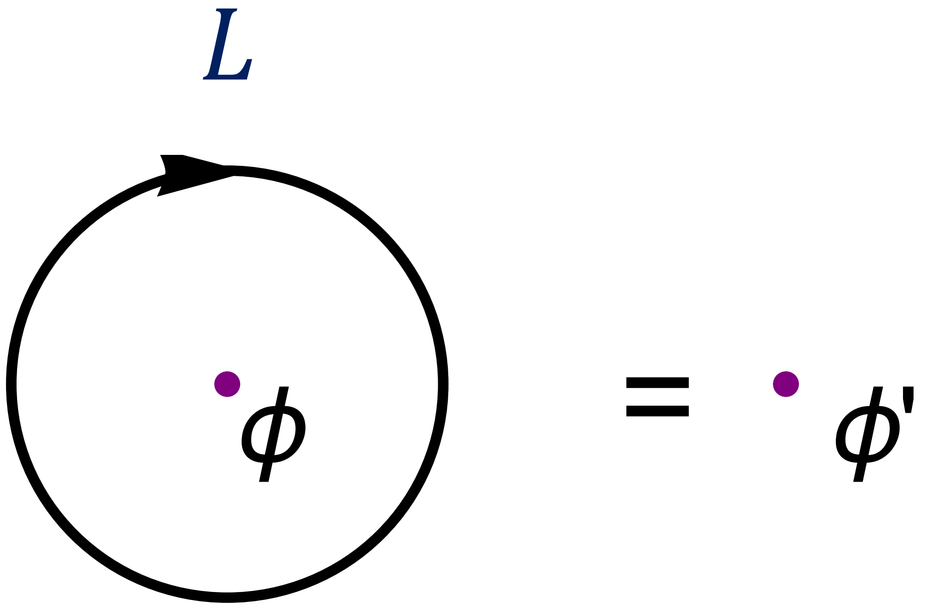


Basic Properties of Topological Lines

3. Action on the Hilbert Space

- Topological line maps a local operator to another of the same conformal weights $(h, \bar{h})$.

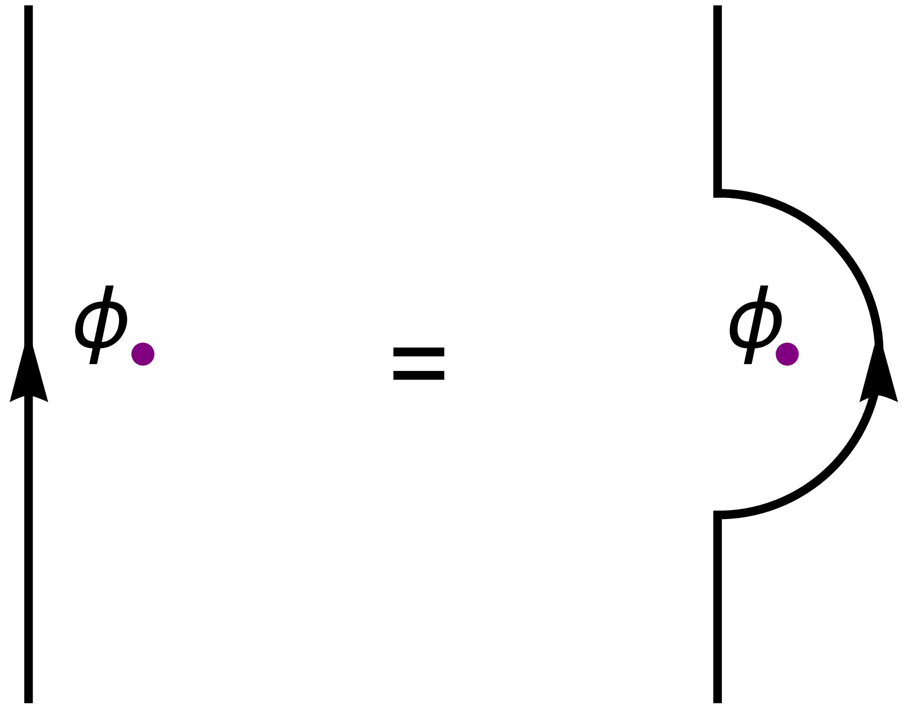
- The map might **not** be **invertible**, in which case L is a non-symmetry line.



Basic Properties of Topological Lines

- In a RG flow, a topological line L is said to be preserved by the relevant deformation $\phi(x)$ if they commute with each other.

- That is, the topological line L doesn't "feel" the insertion of $\phi(x)$ and remains topological after the deformation.

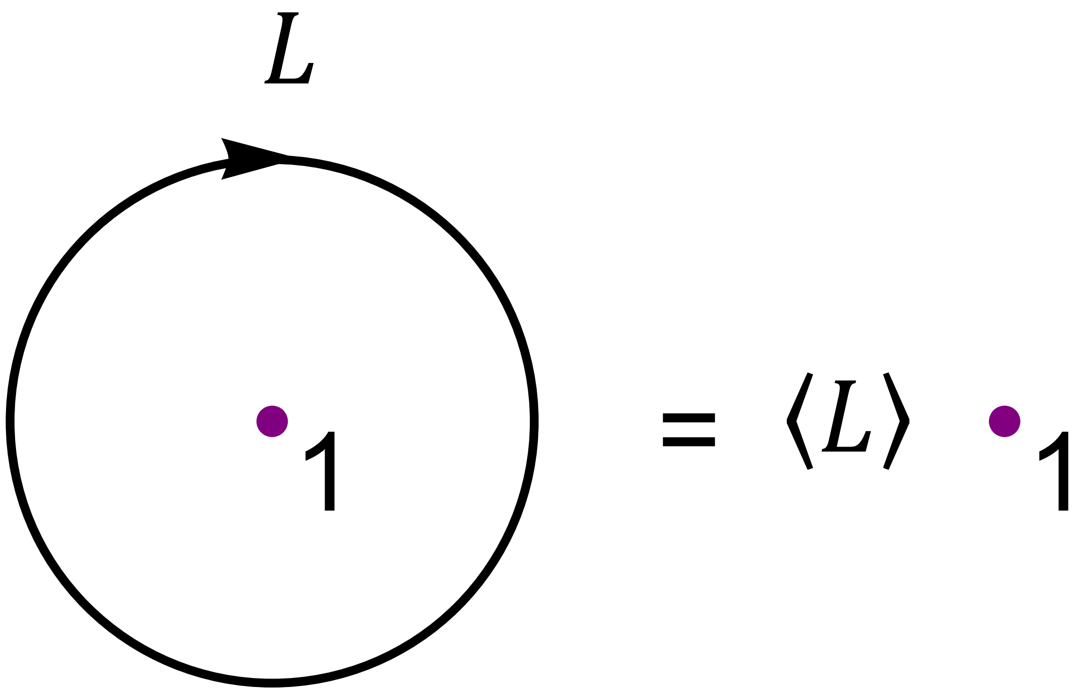


Vacuum Expectation Value $\langle L \rangle$

- Symmetry lines:
  $$\langle L \rangle = 1$$

- Non-Symmetry lines:
  $$\langle L \rangle \neq 1$$

$\langle L \rangle$ is RG invariant!



Assumption: The theory has a unique vacuum $|1\rangle$

Topological Lines in RCFT

- In a diagonal RCFT, there is a simple class of topological lines, the Verlinde lines, that are in 1-to-1 correspondence with the primary operators and commute with both chiral algebras [Verlinde, Petkova-Zuber, Moore-Seiberg].

$$\mathcal{O} \quad \rightarrow \quad L_{\mathcal{O}}$$

Primary $\qquad$ Verlinde Line

- The action on the local operators is determined by the modular S matrix.

Example: 2D Ising Model

- **Three** topological lines in the Ising model [Petkova-Zuber,Frohlich-Fuchs-Runkel-Schweigert…], corresponding to the three primary operators

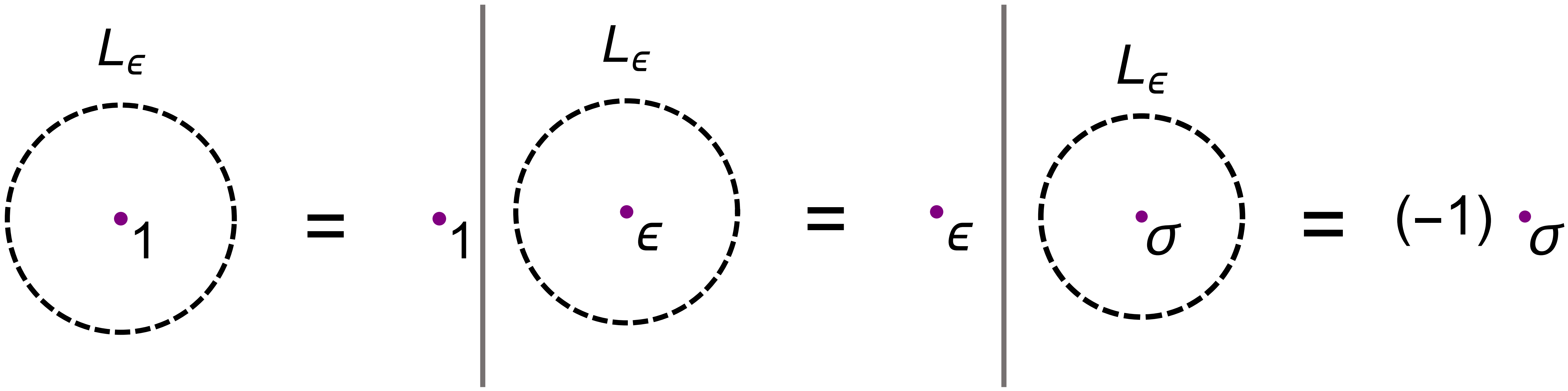
Primary:	$\mathbf{1}_{(0,0)}$	$\varepsilon_{\left(\frac{1}{2},\frac{1}{2}\right)}$	$\sigma_{\left(\frac{1}{16},\frac{1}{16}\right)}$
		$\downarrow$	
Topological Line:	I	L_ε	L_σ

Example: 2D Ising Model

		Symmetry Line?	Vev
Trivial Line	$\mathbb{I}$	Yes	1
$\mathbb{Z}_2$ **Line**	L_ε	Yes	1
Duality Defect	L_σ	No	$\sqrt{2}$

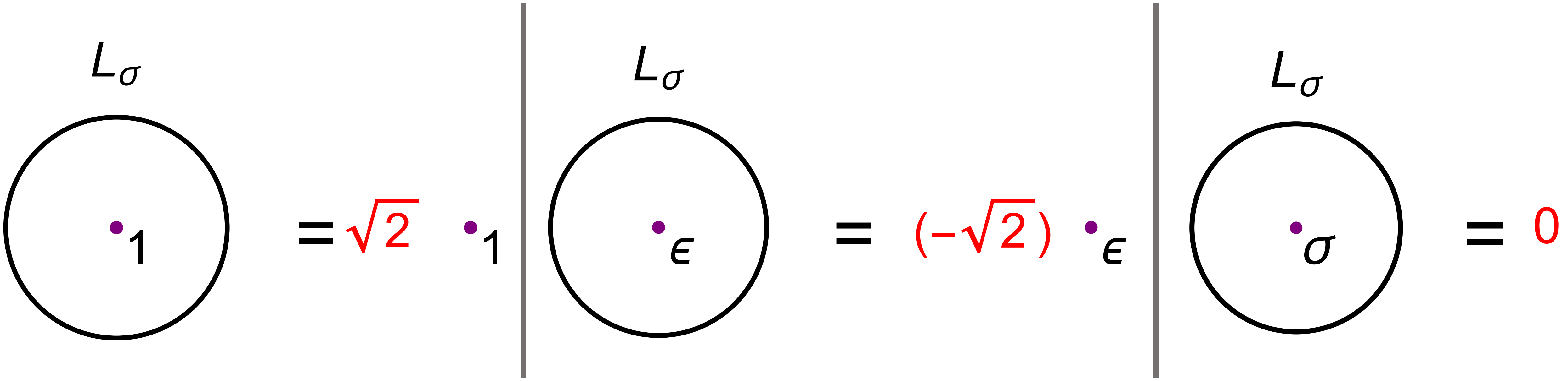
Symmetry Line in the Ising Model

- L_ε implements the $\mathbb{Z}_2$ global symmetry.



Non-Symmetry Line in the Ising Model

- The action of L_σ on local operators is **not invertible**
  → Non-Symmetry line [Petkova-Zuber, Frohlich-Fuchs-Runkel-Schweigert…]



Theorem

- In 2D, if a non-symmetry topological line L with

$$\langle L \rangle \notin \mathbb{Z}_{\geq 0}$$

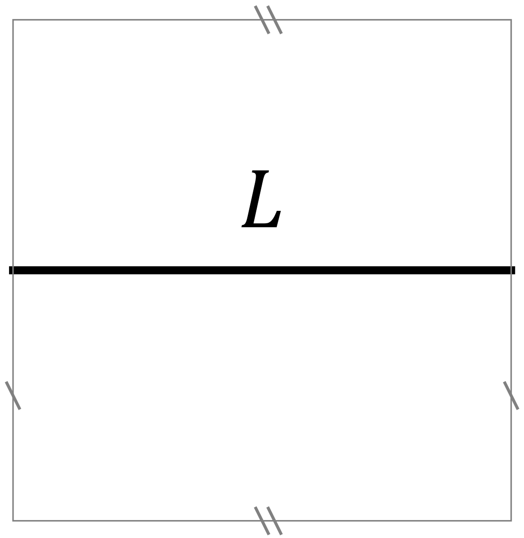
is preserved along a RG flow, then the IR theory can NOT be trivially gapped [Chang-Lin-SHS-Wang-Yin].

Remark:

- Degenerate vacua that are **not** consequences of spontaneous symmetry breaking → Spontaneous Non-Symmetry Breaking
- Similar in spirit to ‘t Hooft anomaly matching: the IR theory has to be nontrivial to match certain degrees of freedom in the UV.

Proof

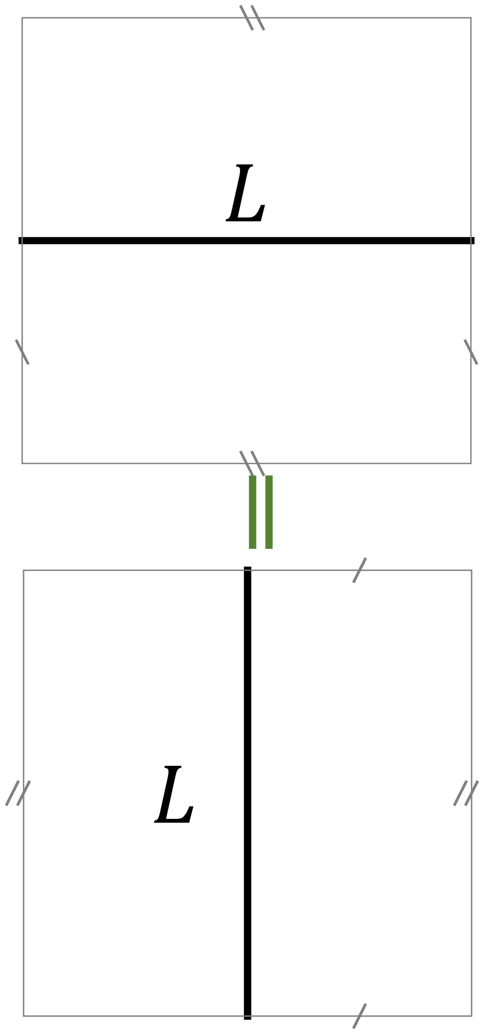
- Prove by contradiction. Assume that the IR theory is trivially gapped, $\mathcal{H}_{IR} = \{1\}$.
- Next, consider the torus partition function of the IR theory with L inserted **horizontally** at a constant time slice.



$$= Tr_{\mathcal{H}_{IR}}\, L = \langle L \rangle$$

$\mathcal{H}_{IR}$

Proof: Modular Invariance



modular invariance

$$= Tr_{\mathcal{H}_{IR}}\, L = \langle L \rangle \notin \mathbb{Z}_{\geq 0}$$

$\mathcal{H}_{IR}$

Contradiction!
Q.E.D.

$$= Tr_{\mathcal{H}_L}\, 1 \in \mathbb{Z}_{\geq 0}$$

$\mathcal{H}_L$

Example: Tri-Critical Ising Model

- Relevant deformation by $\varepsilon'_{(\frac{3}{5},\frac{3}{5})}$ in the $c = \frac{7}{10}$ tri-critical Ising model.
- A non-symmetry topological line of vev $\sqrt{2}$ is preserved along the flow
  $\Longrightarrow$ The IR theory can **NOT** be trivially gapped.

$$\begin{tikzcd} & \textbf{Tri-Critical Ising CFT} \arrow[dl, "-\varepsilon'"'] \arrow[dr, "+\varepsilon'"] & \\ \textbf{Gapped Phase with 3 Vacua} & & \textbf{Ising CFT} \end{tikzcd}$$

[Huse, Zamolodchikov, Kastor-Martinec-Shenker, Cardy-Ludwig, Fateev-Zamolodchikov]

Example: Tri-Critical Ising Model

- Relevant deformation by $\sigma'_{(\frac{7}{16},\frac{7}{16})}$ in the tri-critical Ising model.
- A non-symmetry topological line of vev $\frac{1+\sqrt{5}}{2}$ is preserved along the flow

  $\Longrightarrow$ The IR theory can **NOT** be trivially gapped.
- The $\mathbb{Z}_2$ global symmetry is **explicitly** broken by σ', and yet the vacuum is two-fold degenerate $\rightarrow$ Spontaneous Non-Symmetry Breaking

Tri-Critical Ising CFT

$$\Big\downarrow \sigma'$$

Gapped Phase
with 2 Vacua

[Zamolodchikov, Ellem-Bazhanov]

Conclusion

- **The Power of Symmetry**: Global symmetry, especially when it has ‘t Hooft anomaly, is powerful in constraining RG flows.

- **The Power of Non-Symmetry**: Non-symmetry topological defects are equally powerful in constraining the dynamics of QFT.

- Certain non-symmetry topological lines imply the IR theory can **NOT** be trivially gapped [Chang-Lin-SHS-Wang-Yin].

- Consideration of topological lines and modular invariance sometimes determines the IR TQFT completely.

Outlook

- Given a set of topological lines,
  - Can they be coupled to a non-degenerate vacuum?
  - If so, how many different ways?
  - More generally, is there a TQFT/CFT that realizes these topological lines?

[Moore-Segal, Bhardwaj-Tachikawa]

- Non-symmetry codimension-one topological defects in higher dimensions.
  - For example, the non-symmetry surface defect in the $3d\ U(1)_8$ Chern-Simons theory [Fuchs-Runkel-Schweigert, Kapustin-Saulina].

Thank You!