Boundaries, Interfaces, & Duality in 3d SCFTs

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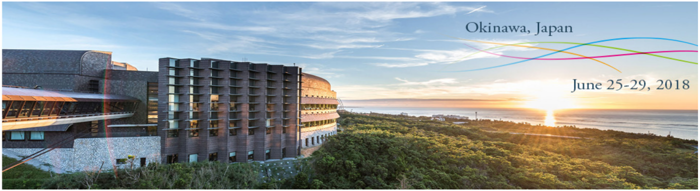


Outline of this talk

Introduction

Boundary Conditions in 3d $\mathcal{N} = 2$ theories

Dual pairs of boundary conditions

Duality interfaces

$(0, 2)$ theories and the 4-simplex

Other Directions

Overview

Motivation:

- There is more to QFT than just local operators. Extended operators can distinguish QFTs, phases thereof...
- Organizing dualities: bulk duality $\curvearrowright$ lower dimensional QFTs supported on extended operators
- SUSY + twisting: equalities among local ops/correlation fns in dual theories can admit precise mathematical reformulations leading to interesting (conjectural) equivalences. What about extended operators?

GOALS:

- Can we enrich and extend bulk (supersymmetric) dualities by adding (half-BPS) defects?

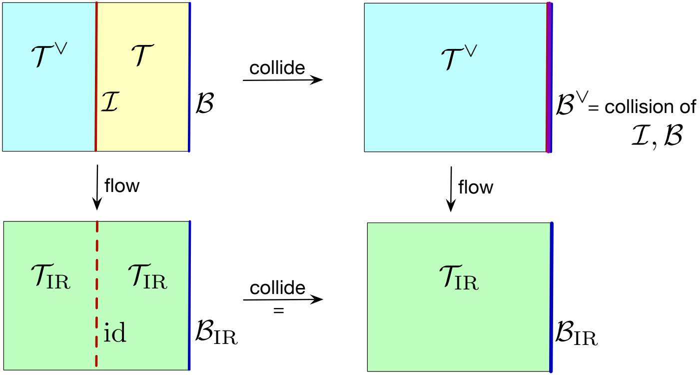
  Today: 3d $\mathcal{N} = 2$ IR (Seiberg-like) dualities, codim-1 objects

- Given such dual pairs, can we understand the resulting equivalences of underlying mathematical objects?

BOUNDARIES AND INTERFACES

- Interface: domain wall between $\mathcal{T}, \mathcal{T}'$
- Boundary: "domain wall" between $\mathcal{T}$, trivial theory

These objects compose in interesting ways.



Overview

Distinct UV Lagrangian theories that flow to common superconformal fixed point

Simple examples to keep in mind:

- SQED/XYZ: $U(1)_0, \Phi, \tilde{\Phi} \leftrightarrow W = XYZ$

  [Aharony-Hanany-Intriligator-Seiberg-Strassler], [de Boer-Hori-Oz]

- SUSY level-rank: $U(N)_{k+N,k} \leftrightarrow SU(k)_{-k-N}$ [Hsin-Seiberg]

Identify UV boundary conditions $\mathcal{B}, \mathcal{B}^\vee$ that flow to the same superconformal $\mathcal{B}_{IR}$

For simplicity, focus on theories with **unitary** gauge groups and **fundamental/antifundamental** matter

Semiclassical bc's in UV YM-CS + matter theories

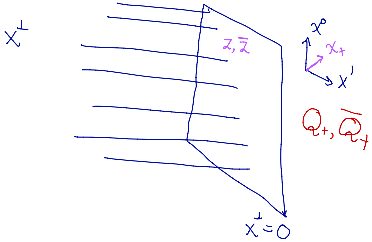
Study (dual pairs of) 3d $\mathcal{N}=2$ theories on a half-space

(expands on [Gadde-Gukov-Putrov])

$\mathbb{R}^{1,1} \times \mathbb{R}_{\leq 0}$

Half-BPS boundary conditions preserving:

- 2d $\mathcal{N}=(0,2)$ SUSY
- 3d $U(1)_R$ (inherited by 2d superalgebra)



$$\{Q_+, \bar{Q}_+\} = -2P_+$$
$$\{Q_-, \bar{Q}_-\} = 2P_-$$
$$\{Q_+, \bar{Q}_-\} = -2i(P_\perp - iZ)$$
$$\{Q_-, \bar{Q}_+\} = 2i(P_\perp + iZ)$$

NB: Our UV computations will fail if emergent IR $U(1)_R$!

Building a boundary

Step One:

Write 3d $\mathcal{N}=2$ multiplets/Lagrangians in 2d $\mathcal{N}=(0,2)$ language

$\rightarrow$

i.e. the 3d theory is a 2d theory on $\mathbb{R}^{1,1}$ whose fields are valued in maps $f:\mathbb{R}\rightarrow$ 3d target

- $\Phi_{3d}\rightarrow\Phi=\phi+\theta^+\psi_+ +\ldots,\Psi=\bar{\psi}_- +\theta^+ f+\ldots$
- $V_{3d}\rightarrow A=\theta^+\bar{\theta}^+ A_+, V_- = A_- +\theta^+\lambda_- +\ldots$
  $S=\sigma+iA_\perp-\theta^+\bar{\lambda}_+\ldots$ (WZ gauge)

Kinetic terms in $x_\perp$ repackaged into $(0,2)$ interactions

Fermionic superpotential, "J-term":

$\int d\theta^+\Psi J(\Phi)=\int d\theta^+\Psi\partial_\perp\Phi\rightarrow f\sim\partial_\perp\bar{\phi}$

NB: Also E-terms, $\bar{D}_+\Psi=E(\Phi), E.J=0$ for SUSY

Step Two:

Identify simple reference boundary conditions, then deform:

$$v|_\partial = 0 \to \left(v + \frac{\partial S_\partial(u)}{\partial u}\right)|_\partial = 0$$

Shortcut: boundary conditions from boundary action e.o.m's:

$$\int_{x^\perp \leq 0} v\partial_\perp u \to \int_{x^\perp \leq 0} v\partial_\perp u + S_\partial(u)$$

Reference b.c's:

Neumann: $\Psi|_\partial = 0 \Rightarrow \partial_\perp \phi|_\partial = 0,\ \psi_-|_\partial = 0$

Dirichlet: $\Phi|_\partial = 0 \Rightarrow \phi|_\partial = 0,\ \psi_+|_\partial = 0$

See e.g. [Witten] for general discussion of elliptic boundary conditions

Rmk: SUSY^c bdy action $\to$ $\perp$ comp. of supercurrent total deriv. on ∂

Step One, cont'd:
Reference b.c's for gauge fields:
Neumann: $F_{\perp\pm}|_\partial = 0, \sigma|_\partial = 0,\ \lambda_+|_\partial = 0$
preserves G gauge symmetry on boundary
(bdy FI term: $\sigma|_\partial = t^\partial$)

Dirichlet: $A, V_-|_\partial = 0 \Rightarrow A_\pm|_\partial = 0,\ D_{3d} - \partial_\perp\sigma|_\partial = 0, \lambda_-|_\partial = 0$
trivializes gauge field at bdy, preserves only G global symmetry

Neumann b.c's require projection onto gauge invariants

Dirichlet b.c's admit boundary monopole operators!

Rmk: Can define bdy conditions defined by singular behavior of bulk fields $\to\ \partial$; simple examples in paper

Step Three:
Couple to extra boundary matter.
Deforms boundary conditions, e.g. Neumann $\to F_{\perp\pm}|_\partial = J^\partial_\pm$

- Neumann b.c. on chirals in presence of $W_{3d}(\Phi)$:
  $\int_{x^\perp \leq 0} dx^\perp \sum_a E_a J^a = W(\Phi)|_\partial$, add $2d$ Fermis Γ s.t.
  $$\sum_i E^i_\Gamma J_{i,\Gamma} = -W(\Phi)|_\partial$$
  choice of *matrix factorization*
- Neumann b.c. on gauge fields

gauge anomaly cancellation

Dual pairs? Match 't Hooft anomalies and flavor symmetries!

Anomaly Interlude

Claim: A 3d fermion with b.c. $\psi_\pm|_\partial = 0$ contributes $\pm\frac{1}{2}\mathbf{f}^2$, i.e. *half* the contribution of a purely 2d fermion, to the anomaly polynomial.

- Add a real mass term $im\epsilon^{\alpha\beta}\psi_{\alpha\beta}$
- Flow to IR
- Match UV-IR anomalies

Two effects:

1. Integrating out massive fermion generates bckgd CS term at level $\frac{1}{2}\text{sign}(m)$
2. Normalizable edge modes (2d fermions) may survive at boundary

Net result:

$\psi_+\big|_\partial = 0$ has total IR anomaly

$$\frac{1}{2}\text{sign}(m) + \left\{ \begin{array}{ll} 0 & \text{if } \ m > 0 \\ 1 & \text{if } \ m < 0 \end{array} \right\} = \frac{1}{2}$$

$\psi_-\big|_\partial = 0$ has total IR anomaly

$$\frac{1}{2}\text{sign}(m) + \left\{ \begin{array}{ll} -1 & \text{if } \ m > 0 \\ 0 & \text{if } \ m < 0 \end{array} \right\} = -\frac{1}{2}$$

NB: detailed (but analogous) computation also for gauginos in paper

Rmk: see also [Moore-Freed] for a geometric avatar

HALF-INDEX

$$II_{\mathcal{T},\mathcal{B}}(x;q) = Tr_{Ops_{\mathcal{B}}}(-1)^R q^{J+R/2} x^e$$

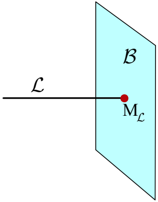
Dual pair of b.c's, same *half-index* (N gauge b.c: [Gadde-Gukov-Putrov]):

- Path integral on $HS^2 \times S^1$ localization for N gauge b.c.: [Yoshida-Sugiyama]
- Operatorial count of $\bar{Q}_+$-closed boundary local operators, *vacuum character* of bdy chiral algebra

$$II_N(x;q) = \prod_{n\geq 0} \frac{1}{1-q^n x} =: \frac{1}{(x;q)_\infty}$$

$$II_D(x;q) = \prod_{n\geq 0} (1-q^{n+1}x^{-1}) = (qx^{-1};q)_\infty$$

Augment with lines to compute characters of other modules:



Overview

Example: Level-rank duality

Proposal (G simple, $k > 0$):

1. $\mathcal{N}=2$ $G_{k+h^\vee}$ YMCS with $\mathcal{D}$ b.c. flows to chiral G_k WZW model coupled to $TFT[G_k]$.
2. $\mathcal{N}=2$ $G_{-k-h^\vee}$ YMCS with $\mathcal{N}$ b.c. MUST be coupled to $\mathcal{T}_{2d}$, flows to $\mathcal{T}_{2d}/G_k$ coupled to $TFT[G_{-k}]$.

$U(N)_{-k-N,-k}$ $\mathcal{N}+k$ fund. Fermis $\leftrightarrow$ $SU(k)_{k+N}$ $\mathcal{D}$

Half-index of $SU(2)_{1+2}$ with Dirichlet boundary conditions on gauge field

$$II_{\mathcal{T}_1,\mathcal{D}} = \chi_0(SU(2)_1) \quad (1)$$

Half-index of $U(1)_{-2}$ with Neumann bc's + 2 fund. Fermis

$$II_{\mathcal{T}_2,\mathcal{N}} = \chi_0(SU(2)_1) \quad (2)$$

Each equality is nontrivial.
Here, equality is a manifestation of level-rank dual *bdy* CFTs!

$$\frac{\text{2 fermions}}{U(1)_2} \simeq SU(2)_1$$

More generally: $\frac{\text{k fermions}}{U(1)_k} \simeq SU(k)_1$

MORE EXAMPLES

3d $\mathcal{N}=2$ dual pairs:

- $U(N), (N_f, \bar{N}_f) \leftrightarrow U(N_f - N), (N_f, \bar{N}_f)$ + singlets, superpotential [Aharony]
- $U(N)_{k+N} \leftrightarrow U(k)_{-k-N}$ (more SUSY level-rank)
- free chiral $\leftrightarrow U(1)_{1/2}$ + chiral $\leftrightarrow U(1)_{-1/2}$ + chiral (SUSY particle-vortex)
- $U(N)_{k+N-\frac{N_f+N_a}{2}}, (N_f, N_a) \leftrightarrow U(k)_{-k-N+\frac{N_f+N_a}{2}}, (N_a, N_f)$ [Benini-Closset-Cremonesi]
- $SU(2)_1$ + adjoint $\leftrightarrow$ free chiral [Jafferis-Yin]

In other examples, many dual pairs we find are the simplest possible choices consistent with:

1. matching boundary flavor symmetries
2. matching boundary 't Hooft anomalies
3. gauge anomaly cancellation
4. superpotential factorization

Boundary chiral algebras

[Costello-Dimofte-Gaiotto]

Dual bdy conditions support $\simeq$ chiral algebras in $\bar{Q}_+$-coh

- $\partial_\perp, \partial_{\bar{z}}$ exact, but ∂_z is not
- "Holomorphic/topological" twist, see [Aganagic-Costello-McNamara-Vafa]
- Bulk ops $\leftrightarrow$ trivial OPE (*except* superpotentials $\rightarrow$ quantum corrections)

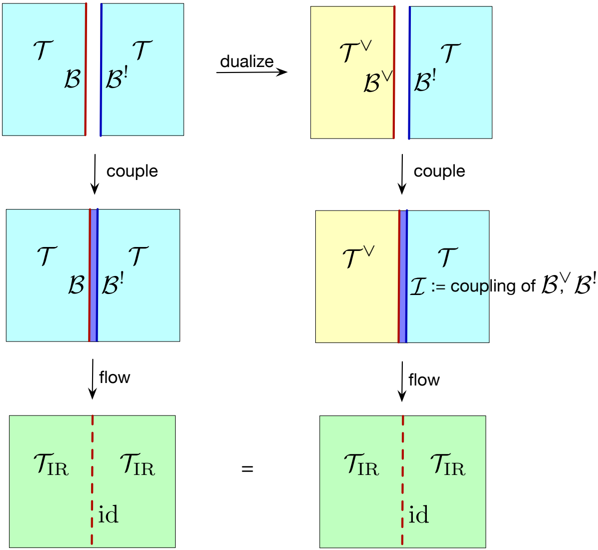
SQED: $(\mathcal{N}NN)+\Gamma$ dual to XYZ: (NDD)

Gauge-invt, $\bar{Q}_+$-closed operators with nontrivial OPE?

$$(\phi\gamma)(z)(\tilde{\phi}\bar{\gamma})(w) \sim \frac{\phi\tilde{\phi}(z)}{(z-w)} \qquad\qquad (\psi_Y)(z)(\psi_Z)(w) \sim \frac{X(z)}{(z-w)}$$

Overview

Build a duality interface



Example

$$U(N)_{-k-N,-k}|\Gamma|SU(k)_{k+N}$$

Neumann b.c's on either side
minimally coupled to Nk free Fermi multiplets Γ
(flavor symmetry from Fermis gauged by coupling to RHS)

$$\Rightarrow$$

Γ form a bifundamental representation of the gauge groups

Can check that total anomaly cancels for $k, N > 0$

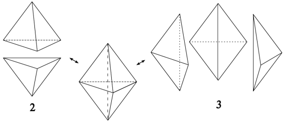
Overview

GEOMETRIC INTERPRETATION?

[Dimofte-NMP], to appear

3d-3d correspondence[Dimofte-Gaiotto-Gukov]

6d $(2,0)$ thy on 3-mfld $M_3 \leftrightarrow$ 3d $\mathcal{N}=2$ theory $\mathcal{T}[M_3]$

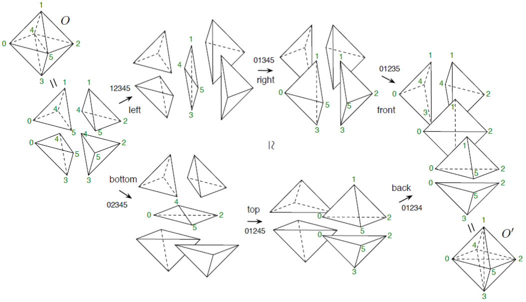
- IR duality of SQED/XYZ $\leftrightarrow$ change of triangulation
- 

2-3 move
- Geometry: Cobordism between two different triangulations = 4-simplex
- Field theory: Change of triangulation implemented by $(0,2)$ duality interface

"Duality" of duality interfaces

- Continue this logic:

  Different ways of gluing 4-simplex together $\leftrightarrow$ Different sequence of duality interfaces



- Different gluings realize homeomorphic 4-simplices $\therefore$ IR equivalence of sequences/collisions of duality interfaces

Associate bdy chiral algebras to simple 4-manifold operation (+ functional identities from half-indices)

see also [Cecotti-Cordova-Vafa, Nakajima, Vafa-Witten, Gadde-Gukov-Putrov, Dedushenko-Gukov-Putrov, Nidaiev-Moore, Feigin-Gukov, ...]

Overview

Many interesting adjacent areas!

1. Similar techniques employed for non-SUSY dual pairs

   [Gaiotto's Nati-Fest talk, Aitken-Baumgartner-Karch-Robinson, Aitken-Karch-Robinson]

2. Consistency of our proposal $\leftarrow$ 2d dualities, e.g. $(0, 2)$ dualities of Gadde-Gukov-Putrov. Generate new $(0, 2)$ dualities?
3. Lift dual b.c's to Seiberg-dual surface operators in 4d $\mathcal{N} = 1$ theories?
4. Techniques apply to 3d theories with more SUSY, half-index useful to study mathematical counterparts

   [Costello-Gaiotto, Aganagic-Okounkov, ...]

5. String/M-theory construction of 3d $\mathcal{N} = 4$ boundary conditions + applications to geometry, rep theory [WIP w/ Dimofte, Hilburn]

⋮

Thank you for your attention!