QED$_3$

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Analyzing a Lagrangian QFT

Semiclassical physics, mostly in the UV (reliable, straightforward, but can be subtle)

- Global symmetry and its ‘t Hooft anomalies
- Weakly coupled limits: flat directions, small parameters, …

Quantum physics in the IR (mostly conjectural)

- Consistency with the global symmetry (including ‘t Hooft anomalies) and the various semiclassical limits
- Approximate methods: lattice, bootstrap, ϵ, $1/N$, …
- Integrability
- String constructions

QED$_3$ [Many references using various methods]

Simple, characteristic example, demonstrating surprising phenomena. Many applications.

- $U(1)$ gauge field a_μ
- N_f fermions ψ^i with charges q_i and masses m_i
- A bare Chern-Simons term.
  - Label the theory as $U(1)_k$ with a parameter k.
  - When all the fermions are massive, at low energies a TQFT $U(1)_{k_{low}}$ with $k_{low} = k + \frac{1}{2}\sum_i \mathrm{sign}(m_i) q_i^2$. Since $k_{low} \in \mathbb{Z}$,

$$k + \frac{1}{2}\sum_i q_i^2 \in \mathbb{Z} .$$

Global symmetries

- Charge-conjugation $\mathcal{C}$: $\quad a_\mu \to -a_\mu$
  (with appropriate action on the fermions)
- $U(1)_0, m = 0$ time-reversal $\mathcal{T}$: $\quad a_0(t,x) \to a_0(-t,x)$
  $$a_i(t,x) \to -a_i(-t,x)$$
  (with appropriate action on the fermions)
- Standard algebra on all the fundamental fields
  $$\mathcal{T}\mathcal{C} = \mathcal{C}\mathcal{T}$$
  $$\mathcal{T}^2 = (-1)^F$$
  $$(\mathcal{C}\mathcal{T})^2 = (-1)^F$$
- For equal charges and masses more symmetries, e.g. $SU(N_f)$.

Global magnetic (topological) symmetry

- $U(1)_M$ symmetry: $j^{\mu} = \frac{1}{2\pi}\epsilon^{\mu\nu\rho}\partial_{\nu}a_{\rho}.$
- The charged operators are monopole operators (like a disorder operator).
  - Remove a point from spacetime and specify boundary conditions around it.
- Massless fermions have zero modes, which can lead to "funny" quantum numbers.

Global magnetic (topological) symmetry

In many applications, the magnetic symmetry is approximate or absent

- In lattice constructions
- When the gauge $U(1)$ is embedded at higher energies in a non-Abelian gauge group
- When the gauge $U(1)$ is emergent
- In the generalization of the gauge $U(1) \cong SO(2)$ to $SO(N)$ with higher N (only a magnetic $\mathbb{Z}_2$ symmetry).

It is natural to break $U(1)_M$ explicitly by adding a monopole operator to the Lagrangian.

Example: $U(1)_k, N_f = 1$ with $q = 1$

$$k \in \mathbb{Z} + \frac{1}{2}$$

- Since k cannot vanish, $\mathcal{T}$ is violated (parity anomaly).
- All gauge invariant polynomials in the fundamental fields are bosons.
- All gauge invariant monopole operators are bosons.
  - $U(1)_{\frac{1}{2}}$ simplest monopole operator has spin 0
  - $U(1)_{\frac{3}{2}}$ simplest monopole operator has spin 1

Example: $U(1)_0, N_f = 2$ with $q = 1$

[Cordova, Hsin, NS]

- All gauge invariant polynomials in the fundamental fields are bosons with integer flavor (isospin) $SU(2)$.
- All gauge invariant monopole operators are bosons with flavor $SU(2)$ isospin $= \frac{M}{2} \bmod 1$.
- On a single monopole $\mathcal{T}^2 = -1$. More generally,

$$\mathcal{T}^2 = (\mathcal{C}\mathcal{T})^2 = (-1)^M$$

rather than the standard $\mathcal{T}^2 = (\mathcal{C}\mathcal{T})^2 = (-1)^F = +1$. Related, but distinct statements in [Wang, T. Senthil; Metlitski, Fidkowski, Chen, Vishwanath; Witten] and see also Hsin's poster.

Examples: $U(1)_0, N_f = 1$ with $q = 2$

[Cordova, Hsin, NS]

- All gauge invariant polynomials in the fundamental fields are bosons.
- All gauge invariant monopole operators have spin $= \frac{M}{2} \bmod 1$, i.e.
$$(-1)^M = (-1)^F$$
  - In the previous example "fractional" isospin. Here "fractional" spin.
- Standard $\mathcal{T}$-symmetry
$$\mathcal{T}^2 = (-1)^F$$

Break the magnetic $U(1)$ symmetry

[Cordova, Hsin, NS; Gomis, Komargodski, NS]

- Add to the Lagrangian a charge 2 monopole operator.
  – Cannot add a charge 1 monopole – it is a fermion.
- It breaks the
  – magnetic symmetry $U(1)_M \to \boldsymbol{Z}_2$: $(-1)^M$
  – $\mathcal{T}$-symmetry
- But it preserves another time-reversal symmetry (a subgroup of the original symmetry)

$$\mathcal{T}' = \mathcal{T} e^{\frac{i\pi}{2}M}$$

Break the magnetic $U(1)$ symmetry

[Cordova, Hsin, NS; Gomis, Komargodski, NS]

$$\mathcal{T}' = \mathcal{T} e^{\frac{i\pi}{2}M}$$

- Its algebra is non-standard

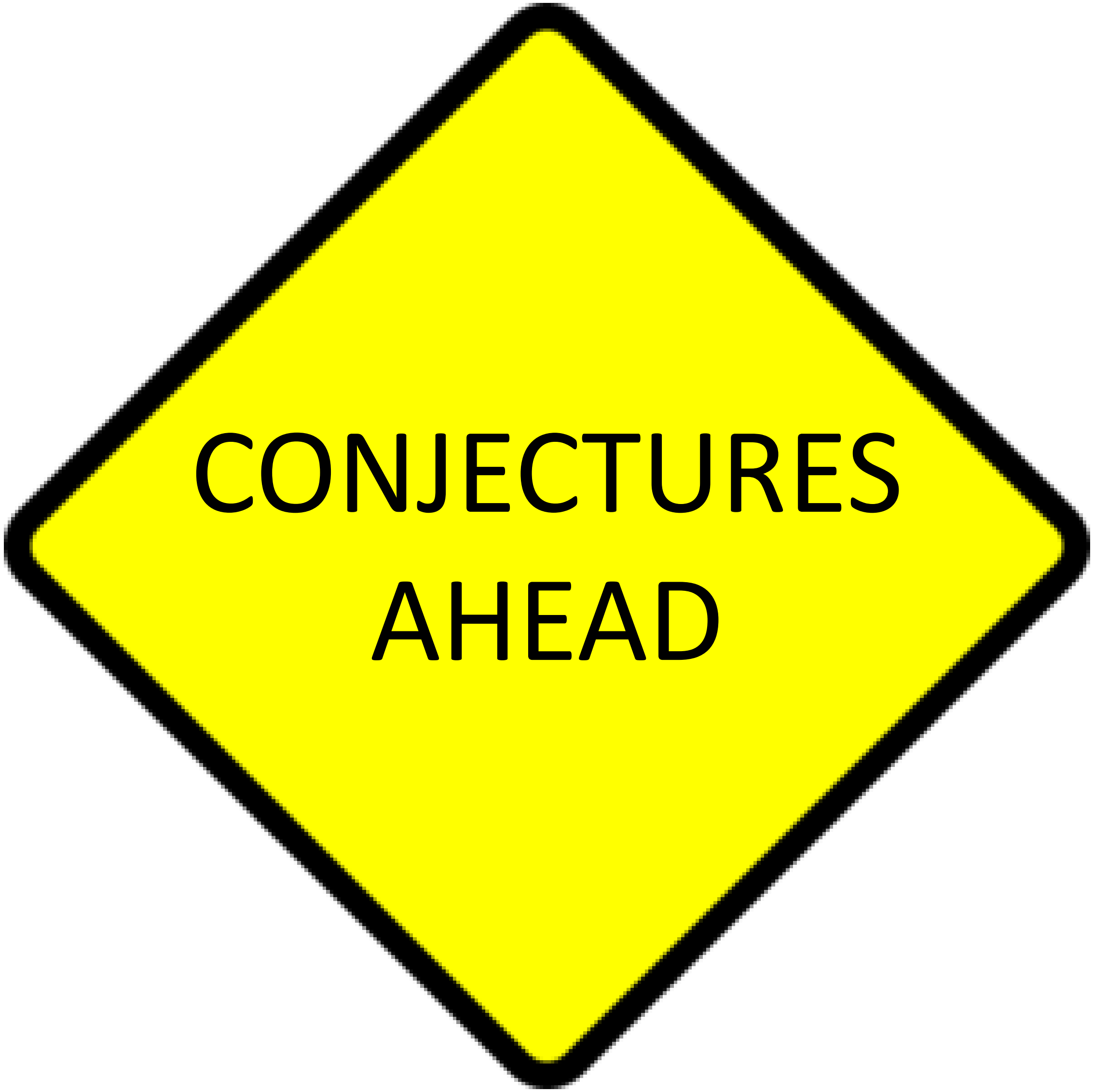
$$\mathcal{T}'\mathcal{C} = \mathcal{C}\mathcal{T}'(-1)^M$$
$$(\mathcal{C}\mathcal{T}')^2 = (-1)^M = (-1)^F$$
$$(\mathcal{T}')^2 = 1 \neq (-1)^F$$

$\mathcal{C}$ and $\mathcal{T}'$ do not commute.

$\mathcal{T}'$ is not a conventional time-reversal symmetry, but $\mathcal{C}\mathcal{T}'$ is.

What is the long distance behavior?

- What are the phases?
  - – Gapped? Topological?
  - – Symmetry breaking?
- What happens at the phase transitions? First or second order?
  - – And if second order, free or interacting?
- It is clear for large $|m|$


CONJECTURES
AHEAD

Vary m at large $|k|$ or at large N_f

- Large k. Essentially free fermions with a modified Gauss law constraint
- Large N_f. Second-order transition as a function of the fermion mass m [Appelquist, Nash, Wijewardhana]
- Conjecture that this is the case for all k, q, and N_f.

$U(1)_{k-\frac{1}{2}N_f q^2}$ $\qquad$ $U(1)_{k+\frac{1}{2}N_f q^2}$

$m < 0$ $\qquad$ $m > 0$

Second order transition at $m = 0$

$U(1)_k, N_f = 1$ with charge one ($q = 1$)

$U(1)_{1/2}$ flows to the $O(2)$ Wilson-Fisher fixed point
[Chen, Fisher, Wu; Barkeshli, McGreevy; NS, T. Senthil, Wang, Witten; Karch, Tong]

- The spin-zero monopole operator of the gauge theory is the order parameter of the $O(2)$ model.
- Emergent $\mathcal{T}$-symmetry in the IR

$U(1)_{3/2}$ flows to a fixed point with $SO(3)$ symmetry
[Aharony, Benini, Hsin, NS; Benini, Hsin, NS]

- The spin-one monopole operator of the gauge theory becomes a conserved current in $\mathrm{O}(2) \rightarrow SO(3)$.
- Several dual bosonic and fermionic descriptions

$U(1)_0, N_f = 2$ with $q = 1$

[Xu, You; Karch, Tong; Hsin, NS; Benini, Hsin, NS; Wang, Nahum, Metlitski, Xu, T. Senthil]

- Microscopically: global $U(2)$, $\mathcal{C}$ (they do not commute), and $\mathcal{T}$-symmetry
  - $\mathcal{T}$ and $\mathcal{CT}$ have a non-standard algebra

$$\mathcal{T}^2 = (\mathcal{CT})^2 = (-1)^M$$

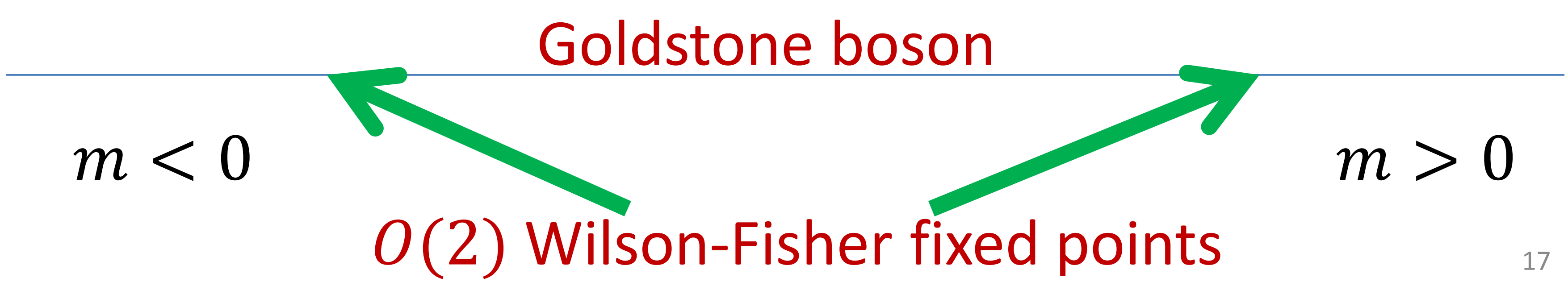
- Conjectured IR behavior: fixed point with enhanced global $O(4)$ symmetry
- Dual fermionic description: $U(1)_0$ with $N_f = 2$
  - Its $SU(2)$ flavor symmetry includes the original $U(1)_M$, and vice versa.

$U(1)_0, N_f = 2$ with $q = 1$

[Benini, Hsin, NS; Komargodski, NS]

Break the magnetic symmetry by adding to the Lagrangian a double monopole operator

- This explicitly breaks the flavor $SU(2) \to U(1)$ and the magnetic $U(1)_M \to \mathbb{Z}_2$.
- For some range of m the remaining flavor $U(1)$ symmetry is spontaneously broken.
  - The UV theory is massive, but the IR theory is gapless!



$U(1)_0, N_f = 1$ with $q = 2$

[Cordova, Hsin, NS]

- Flows to a free Dirac fermion χ and a decoupled $U(1)_2$ TQFT
  - Similar, but not identical to [Son; Wang, T. Senthil; Metlitski, Vishwanath; NS, T. Senthil, Wang, Witten].
- The monopole operator in the UV becomes the free fermion χ in the IR.
- The Wilson line with charge 1 in the UV is described as the Wilson line of $U(1)_2$ (semion).

Break the magnetic $U(1)$ symmetry

[Cordova, Hsin, NS; Gomis, Komargodski, NS]

Add to the Lagrangian a charge 2 monopole operator.

- It preserves $\mathcal{T}' = \mathcal{T} e^{\frac{i\pi}{2}M}$ with a nonstandard algebra
- In the IR it splits the fixed point with a massless Dirac fermion to two points with massless Majorana fermions

$U(1)_2$ $\qquad\qquad\qquad$ $U(1)_2$ $\qquad\qquad\qquad$ $U(1)_2$

$m < 0$ $\qquad\qquad\qquad\qquad\qquad\qquad$ $m > 0$

Massless Majorana fermions

Summary

QED$_3$: $U(1)_k$ gauge theory with charged fermions.

- Subtle global symmetry, especially in the action on monopoles
  - Fractional (global symmetry) charges and spins
  - Unusual algebras involving $\mathcal{C}$ and $\mathcal{T}$
- Conjectures about the IR behavior of these systems
  - Enhanced global symmetry
  - Dual descriptions

This is a tiny part of a long story about the dynamics of 2+1 dimensional QFT.

Most of it is still unknown…