Line Defect Schur Indices, Verlinde Algebras and $U(1)_r$ fixed points

Fei Yan

University of Texas at Austin

Strings 2018
Okinawa Institute of Science and Technology Graduate University

Joint work with Andrew Neitzke
arXiv: 1708.05323

June 25, 2018

Outlook

An interesting feature of the line defect Schur index in 4d $\mathcal{N} = 2$ theories ([Dimofte-Gaiotto-Gukov][Córdova-Gaiotto-Shao]):

Outlook

An interesting feature of the line defect Schur index in 4d $\mathcal{N} = 2$ theories ([Dimofte-Gaiotto-Gukov][Córdova-Gaiotto-Shao]):

There is a close relation between

- Schur index in the presence of a supersymmetric (half) line defect L,
- the vevs $\langle L \rangle$ in $U(1)_r$-invariant vacua of the theory compactified on S^1.

Schur Index and Chiral Algebra

- 4d $\mathcal{N} = 2$ SCFT $\mathcal{T} \longrightarrow$ 2d chiral algebra $\mathcal{A}$ (Beem-Lemos-Liendo-Peelaers-Rastelli-van Rees)

- Local operators contributing to the Schur index correspond to operators in the vacuum module of $\mathcal{A}$.

$$\mathcal{I}(q, a_i) = \chi_0(q, a_i).$$

Schur Index and Chiral Algebra

- 4d $\mathcal{N} = 2$ SCFT $\mathcal{T} \longrightarrow$ 2d chiral algebra $\mathcal{A}$ (Beem-Lemos-Liendo-Peelaers-Rastelli-van Rees)

- Local operators contributing to the Schur index correspond to operators in the vacuum module of $\mathcal{A}$.

$$\mathcal{I}(q, a_i) = \chi_0(q, a_i).$$

- What about characters of non-vacuum modules of $\mathcal{A}$?

Line Defect Schur Index

Consider supersymmetric half line defect L:

- extends along a ray in $\mathbb{R}^4$ and terminates at a point.
- The endpoint support local opeators $\rightarrow$ count via the line defect Schur index ([Dimofte-Gaiotto-Gukov],[Córdova-Gaiotto-Shao]):

$$\mathcal{I}_L(q, a_i) = \mathrm{Tr}_{\mathcal{H}'}(-1)^F q^{\Delta - R} \prod_i a_i^{f_i}.$$

Line Defect Schur Index and Verlinde Algebra

In various examples Córdova-Gaiotto-Shao found that:

$$\mathcal{I}_L(q) = \sum_{\beta} v_{L,\beta}(q)\chi_\beta(q),$$

β runs over a finite collection of modules closed under modular S action.

Line Defect Schur Index and Verlinde Algebra

In various examples Córdova-Gaiotto-Shao found that:

$$\mathcal{I}_L(q) = \sum_\beta v_{L,\beta}(q)\chi_\beta(q),$$

β runs over a finite collection of modules closed under modular S action. We can use the Verlinde formula to define a commutative associative algebra $\mathcal{V}$ generated by $[\Phi_\beta]$, with

$$[\Phi_\alpha] \times [\Phi_\beta] = c^\gamma_{\alpha\beta}[\Phi_\gamma].$$

To every line defect L one could associate $f(L) = \sum_\beta v_{L,\beta}(q=1)[\Phi_\beta] \in \mathcal{V}$.

Line Defect Schur Index and Verlinde Algebra

In various examples Córdova-Gaiotto-Shao found that:

$$\mathcal{I}_L(q) = \sum_{\beta} v_{L,\beta}(q)\chi_\beta(q),$$

β runs over a finite collection of modules closed under modular S action.
We can use the Verlinde formula to define a commutative associative algebra $\mathcal{V}$ generated by $[\Phi_\beta]$, with

$$[\Phi_\alpha] \times [\Phi_\beta] = c^{\gamma}_{\alpha\beta}[\Phi_\gamma].$$

To every line defect L one could associate $f(L) = \sum_{\beta} v_{L,\beta}(q=1)[\Phi_\beta] \in \mathcal{V}$.

$f : \mathcal{L} \to \mathcal{V}$, where $\mathcal{L}$ is the commutative OPE algebra of line defects.
There is evidence that f is a homomorphism.
f is very forgetful.

$U(1)_r$-fixed Locus in 3d

- Compactifying a 4d $\mathcal{N} = 2$ theory on S^1, the Coulomb branch of the compactified theory is hyperkähler [Seiberg-Witten].
- For theories of class $S[\mathfrak{g}, C]$, get moduli space $\mathcal{M}(\mathfrak{g}, C)$ of solutions to Hitchin's equations on C with gauge algebra $\mathfrak{g}$ [Gaiotto-Moore-Neitzke].
- $U(1)_r$ symmetry $\rightarrow$ Hitchin action on $\mathcal{M}(\mathfrak{g}, C)$ [Hitchin].
- For Argyres-Douglas theories, the $U(1)_r$-fixed locus consists of finitely many isolated points. Moreover they are in 1-to-1 correspondence with certain highest-weight modules of $\mathcal{A}$.

  [Fredrickson-Neitzke],[Fredrickson-Pei-W.Yan-Ye]

$U(1)_r$-fixed Locus in 3d

How are the $U(1)_r$ fixed points related to $\mathcal{V}$ and $\mathcal{L}$?

$U(1)_r$-fixed Locus in 3d

How are the $U(1)_r$ fixed points related to $\mathcal{V}$ and $\mathcal{L}$?

- Denote $\mathcal{O}(F)$ as the algebra of functions on the set of $U(1)_r$-fixed points F. The modular S operator canonically diagonalizes the Verlinde algebra $\mathcal{V}$. There is a canonical isomorphism

$$h : \mathcal{V} \to \mathcal{O}(F).$$

- Consider the vacuum expectation values of line defects wrapped around S^1, these vevs are functions on the Hitchin moduli space. Restricting these vevs to F gives a homomorphism

$$g : \mathcal{L} \to \mathcal{O}(F).$$

g is also very forgetful.

A Commutative Diagram

$$\begin{tikzcd} \mathcal{L} \arrow[r, "f"] \arrow[dr, "g"'] & \mathcal{V} \arrow[d, "h"] \\ & \mathcal{O}(F) \end{tikzcd}$$

- $\mathcal{L}$: OPE algebra of line defects
- $\mathcal{V}$: Verlinde-like algebra associated to the 2d chiral algebra $\mathcal{A}$
- $\mathcal{O}(F)$: Algebra of functions on $U(1)_r$-fixed locus F.

Thank You!