Scrambling and Relative entropy

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Scrambling

Scrambling refers to the phenomena of quick delocalization of quantum information in thermal states. [Sekino Susskind] [Lashkari Stanford et al]....

The typical time scale of these phenomena is the scrambling time $t_{sc} = \beta \log c$

In a chaotic system, due to the delocalization, the thermal RDM ρ_β and its perturbation $W^\dagger(t)\rho_\beta W(t)$ by a local operator W(t) become indistinguishable in any local subregion A after the scrambling time.

Scrambling(2)

The distance between the thermal RDM ρ_β and its perturbation $W^\dagger(t)\rho_\beta W(t)$ is suitable to characterize this delocalization ie, scrambling.

The decay rate of the distance characterizes the chaotic nature of the system.

The distance between two density matrices is measured by relative entropy.

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The distance between two density matrices is measured by relative entropy.

Motivated by this observation, we studied the dynamics of scrambling using the relative entropy in 2d large c CFTs as well as several spin chain models.

Relative entropy

- Relative entropy between two density matrices is

$$S(\rho||\sigma) = \mathrm{tr}\,(\rho \log \rho) - \mathrm{tr}\,(\rho \log \sigma)$$

- This quantity measures the distance between the two.

- Is Positive definite.

- Is zero iff $\rho = \sigma$.

Set up.

We start from a TFD state, and perturb it
By a local operator W(t).

$$|\Psi_1\rangle = \quad |TFD\rangle = \frac{1}{\sqrt{Z(\beta)}}\sum_n e^{-\frac{\beta}{2}E_n}|E_n\rangle_L|E_n\rangle_R$$

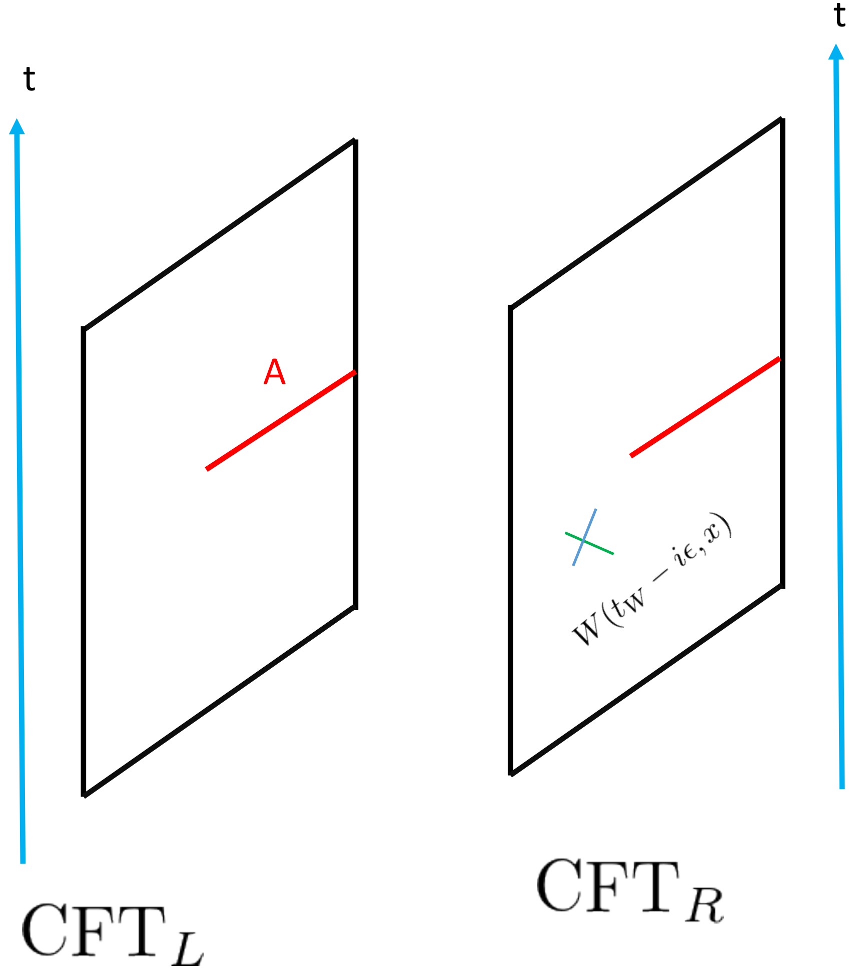
$$|\Psi_2\rangle = W(t_W - i\epsilon, x)|TFD\rangle$$

Evolve both CFTs forward in time.

Subsystem A: half spaces in both CFTs

Relative entropy between two RDMS.

$$S(\rho_1||\rho_2)$$



The relative entropy in the large C limit

- We first computed the relative entropy in the large C limit, where we can use the gravity description, as well as the RT formula.

- When $t \gg \log \frac{\beta^2}{\epsilon}$ the relative entropy decays exponentially,

$$S(\rho_1||\rho_2) \sim ce^{-\frac{t}{\beta}}$$

- The relative entropy becomes O(1) at the scrambling time $t_{sc} = \beta \log c$. After this,the large c approximation breaks down, and quantum corrections to the RT formula become important, in order to further follow the time evolution.

Some Numerics in spin chain models.

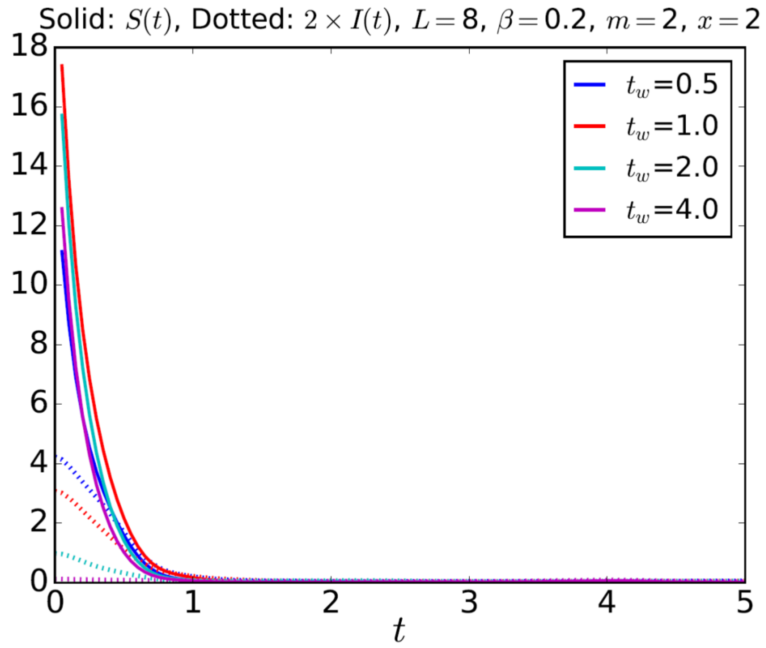
In order to further follow the time evolution of the relative entropy after the scrambling time, we performed numerical calculations in several spin chain models.

$$H = -\sum_{i=1}^{N} (Z_i Z_{i+1} + gX_i + hZ_i)$$

The Hamiltonian is integrable when $h = 0$.

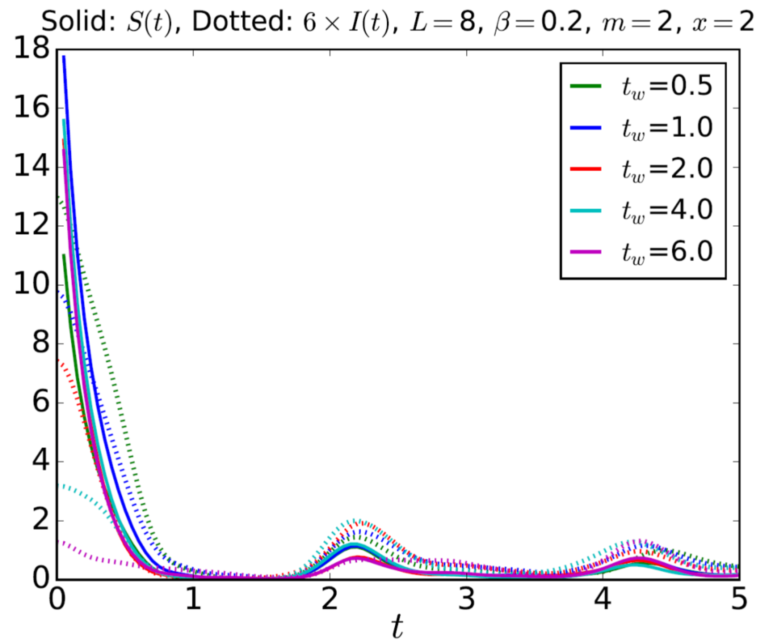
A chaotic point (g,h)= (-1.05, 0.5).

Non integrable case



No oscillations in the late time regime

Integrable case



Oscillations in the late time regime

In non chaotic systems, we observe recursions of the relative entropy , however in chaotic systems, there are no recursions.

Conclusions

- I explained the application of relative entropy to the physics of scrambling .

- Can we generalize the computation to higher dimensions?

- Probably the holographic relative entropy decays fastest. Can we show

This ?

Thank you!