Bit thread decomposition of holographic entanglement

Bogdan Stoica

Martin A. Fisher School of Physics
Brandeis University

25 June 2018

S. X. Cui, P. Hayden, T. He, M. Headrick, B. Stoica, M. Walter

Motivation: Gravity and entanglement

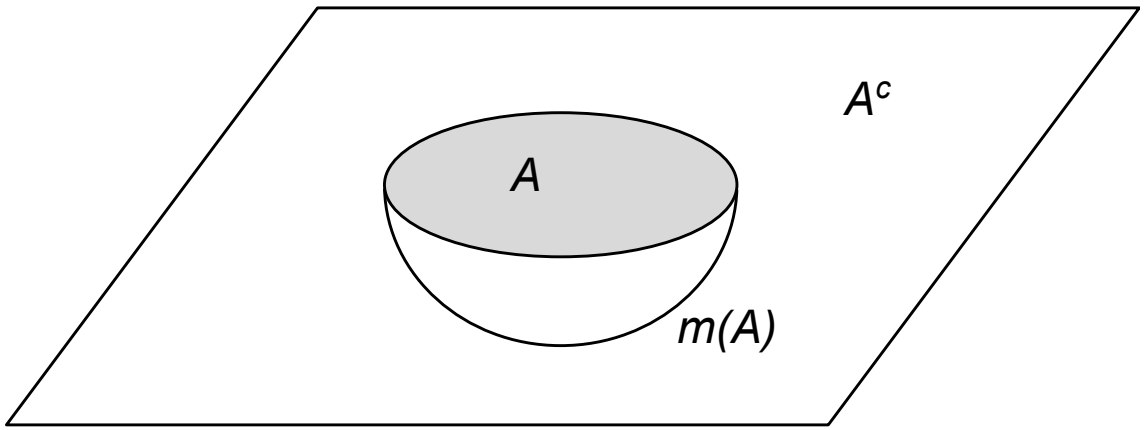
- Recent developments at the intersection of gravity, field theory and quantum information suggest that the connection between gravity and entanglement is essential for understanding how gravitational degrees of freedom emerge from quantum ones.

- How does geometry arise from information? What mathematical results can be uncovered along the way? What other ingredients are missing?

(Holographic) entanglement entropy

- In quantum systems, *entanglement entropy* is a measure of the correlation of spatial region A with its complement A^c. Hard to compute in field theory!
- For quantum field theory states with smooth geometric duals, entanglement entropy has a holographic prescription as the bulk minimal area $m(A)$ homologous to the boundary region A,

$$S(A) = \frac{m(A)}{4G_N}. \qquad \text{[Ryu-Takayanagi]}$$



(Qu)Bit threads

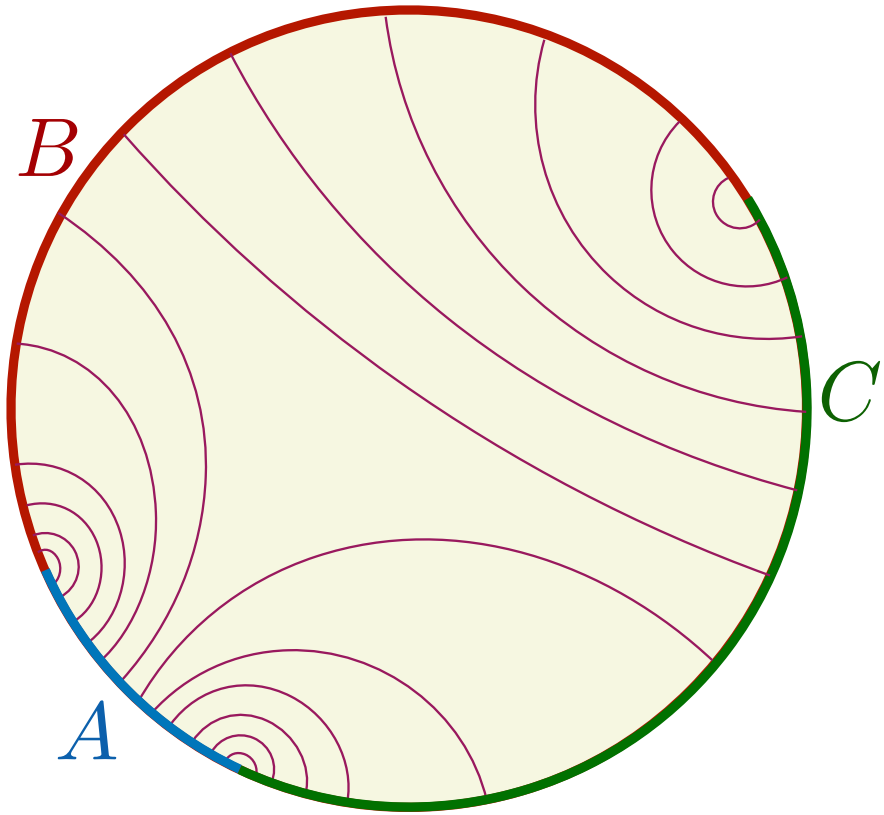
- A local prescription for entanglement entropy?
- Imagine threads of entanglement through the bulk, connecting any two Planck-area regions on the boundary. These are bit threads.
- *Divergenceless bulk vector flows of bounded norm*.
- Entanglement entropy between A and A^c: *Maximum number of threads passing through the bulk from A to A^c.*
- The bottleneck constraining the maximum number of flows between A and A^c is the bulk minimal area $m(A)$ ([Ryu-Takayanagi] recovered).

[Freedman-Headrick] [Headrick–Hubeny]

Entanglement from (Qu)Bit threads

Key insight

Properties of bit thread flows $\Leftrightarrow$ Properties of HEE



Concurrent flows theorem

- Holographic entropic inequalities follow from theorems on flows.

Theorem

Any bulk with n boundary regions A_i, there exist $n(n-1)/2$ pairwise flows v_{ij}, $i<j$, between regions A_i and A_j, such that:

- The flows for region A_i realize its entanglement entropy,

$$S(A_i) = \sum_{j \neq i} \int_{A_i} v_{ij}.$$

- Flows v_{ij} can be concurrently realized on the manifold.

Corollary

The monogamy of holographic mutual information holds.

Entanglement decomposition

Decomposition theorem

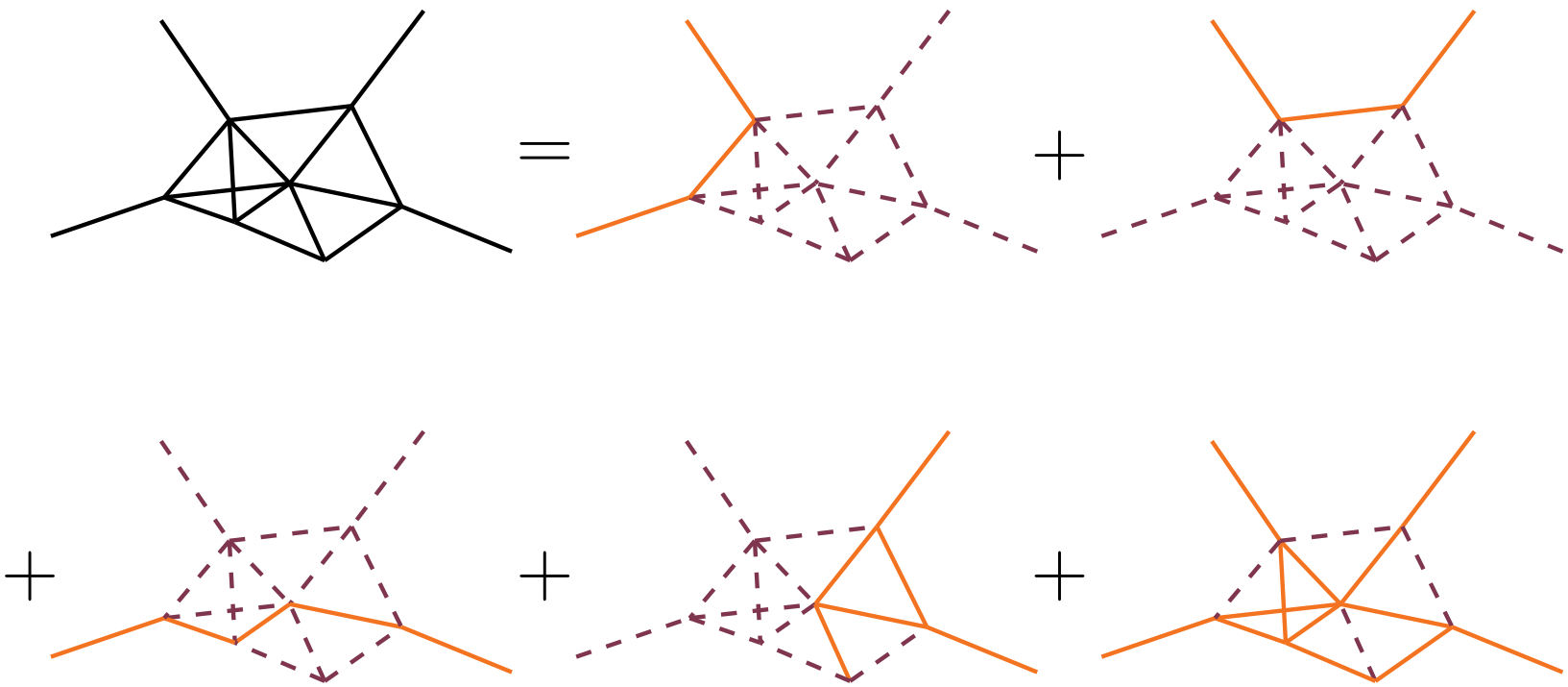
The entanglement of three boundary regions can be decomposed into the entanglement of three Bell pairs.

Decomposition conjecture

The entanglement of four boundary regions can be decomposed into the entanglement of six Bell pairs, plus a remainder four-region perfect tensor entanglement.

- The conjecture implies the monogamy of mutual information: The six Bell pairs have vanishing contribution to MMI, the four-region perfect tensor contributes positively.

Graph decomposition conjecture



Thank you!