The Schwarzian and black hole physics

Thomas Mertens

Ghent University

Based on arXiv:1606.03438 with J. Engelsöy and H. Verlinde
arXiv:1705.08408 with G.J. Turiaci and H. Verlinde
arXiv:1801.09605
arXiv:1804.09834 with H. Lam, G.J. Turiaci and H. Verlinde
arXiv:1806.07765 with A. Blommaert and H. Verschelde

Motivation

Schwarzian theory: $S_{Schw} = -C \int dt \, \{f, \tau\}$ where
$\{f, \tau\} = \frac{f'''}{f'} - \frac{3}{2}\left(\frac{f''}{f'}\right)^2$ the Schwarzian derivative

Motivation

Schwarzian theory: $S_{Schw} = -C \int dt \, \{f, \tau\}$ where
$\{f, \tau\} = \frac{f'''}{f'} - \frac{3}{2}\left(\frac{f''}{f'}\right)^2$ the Schwarzian derivative

Appears in:

- SYK model at low energies
  Schwarzian theory describes regime of $N \to \infty$ and $\beta J \to \infty$
- Jackiw-Teitelboim (JT) 2d dilaton gravity
  JT is holographically dual to Schwarzian theory
  $\to$ Dynamics of wiggly boundary curve described by S_{Schw}
  Compare to CS / WZW, 3d gravity / Liouville topological dualities

Definition

Main goal:
Compute all correlation functions:

$$\left\langle \mathcal{O}_{\ell_1}\mathcal{O}_{\ell_2}\dots\right\rangle_\beta = \frac{1}{Z}\int_{\mathcal{M}}[\mathcal{D}f]\mathcal{O}_{\ell_1}\mathcal{O}_{\ell_2}\dots\ e^{C\int_0^\beta d\tau\left(\left\{f,\tau\right\}+\frac{2\pi^2}{\beta^2}f'^2\right)}$$

- with $\mathcal{M} = \mathrm{Diff}(S^1)/SL(2,\mathbb{R}), \qquad f(\tau+\beta) = f(\tau)+\beta$

Definition

Main goal:
Compute all correlation functions:

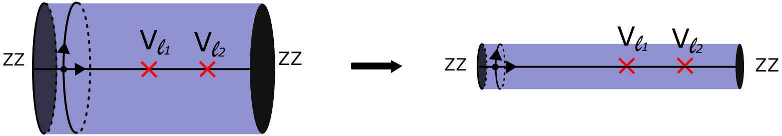
$$\langle \mathcal{O}_{\ell_1} \mathcal{O}_{\ell_2} \dots \rangle_\beta = \frac{1}{Z} \int_{\mathcal{M}} [\mathcal{D} f] \mathcal{O}_{\ell_1} \mathcal{O}_{\ell_2} \dots \; e^{C \int_0^\beta d\tau \left(\left\{ f, \tau \right\} + \frac{2\pi^2}{\beta^2} f'^2 \right)}$$

- with $\mathcal{M} = \mathrm{Diff}(S^1)/SL(2,\mathbb{R})$, $\qquad f(\tau + \beta) = f(\tau) + \beta$
- Bilocal operators: $\mathcal{O}_\ell(\tau_1, \tau_2) = \left(\frac{f'(\tau_1) f'(\tau_2)}{\frac{\beta}{\pi} \sin^2 \frac{\pi}{\beta} [f(\tau_1) - f(\tau_2)]} \right)^\ell$

$C \to +\infty$ is probing semi-classical regime: gravitational physics in exact correlators

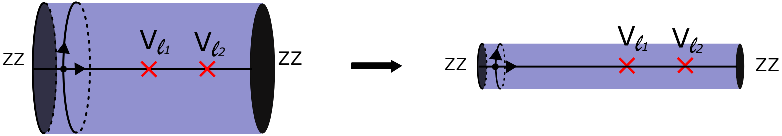
Method: 2d Liouville CFT

Consider 2d Liouville CFT on a cylinder between ZZ-branes



Method: 2d Liouville CFT

Consider 2d Liouville CFT on a cylinder between ZZ-branes

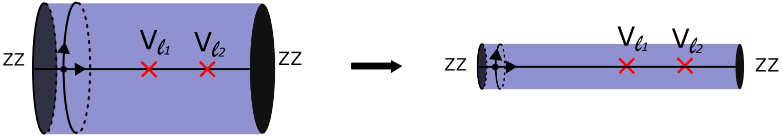


Take Schwarzian double-scaling limit:

- $c \to +\infty$
- radius $\to 0$

$, c \times$ radius $\sim C$, fixed

Consider 2d Liouville CFT on a cylinder between ZZ-branes



Take Schwarzian double-scaling limit:

- $c \to +\infty$
- radius $\to 0$

$, c \times$ radius $\sim C,$ fixed

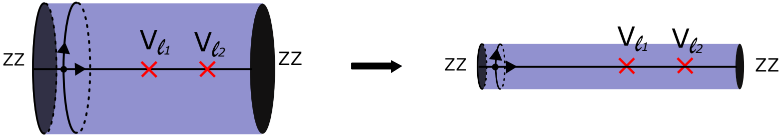
Path-integral proof of link Liouville - Schwarzian

Liouville-Schwarzian Dictionary:

- $T(w) \;\to\; -\frac{c}{24\pi}\left\{F(\sigma),\sigma\right\}, \qquad F \equiv \tan\left(\frac{\pi f(\tau)}{\beta}\right)$
- $V_\ell \equiv e^{2\ell\phi} \;\to\; \mathcal{O}_\ell(\tau_1,\tau_2) \equiv \left(\frac{F_1' F_2'}{(F_1 - F_2)^2}\right)^\ell$

Method: 2d Liouville CFT

Consider 2d Liouville CFT on a cylinder between ZZ-branes



Take Schwarzian double-scaling limit:

- $c \to +\infty$
- radius $\to 0$

, $c \times$ radius $\sim C$, fixed

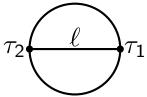
Path-integral proof of link Liouville - Schwarzian

Liouville-Schwarzian Dictionary:

- $T(w) \;\to\; -\frac{c}{24\pi}\{F(\sigma),\sigma\}, \qquad F \equiv \tan\left(\frac{\pi f(\tau)}{\beta}\right)$
- $V_\ell \equiv e^{2\ell\phi} \;\to\; \mathcal{O}_\ell(\tau_1,\tau_2) \equiv \left(\frac{F_1' F_2'}{(F_1-F_2)^2}\right)^\ell$

Liouville amplitudes with V_ℓ's between ZZ's $\to$ Schwarzian bilocal correlators

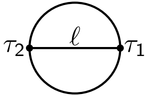
Schwarzian diagram:

$\langle \mathcal{O}_\ell(\tau_1, \tau_2) \rangle =$ 

Result: 2-point correlator

Schwarzian diagram:

$$\langle \mathcal{O}_\ell(\tau_1,\tau_2) \rangle =$$



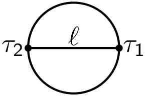
$$= \frac{1}{Z(\beta)} \int d\mu(k_1) d\mu(k_2)\, e^{-\tau k_1^2 - (\beta-\tau) k_2^2}\, \frac{\Gamma\left(\ell \pm i(k_1 \pm k_2)\right)}{2\sqrt{\pi}\Gamma(2\ell)}$$

$$d\mu(k) \equiv dk^2 \sinh(2\pi k)$$

Result: 2-point correlator

Schwarzian diagram:

$$\langle \mathcal{O}_\ell(\tau_1, \tau_2) \rangle =$$



$$= \frac{1}{Z(\beta)} \int d\mu(k_1) d\mu(k_2)\, e^{-\tau k_1^2 - (\beta - \tau) k_2^2}\, \frac{\Gamma\left(\ell \pm i(k_1 \pm k_2)\right)}{2\sqrt{\pi}\Gamma(2\ell)}$$

$$d\mu(k) \equiv dk^2 \sinh(2\pi k)$$

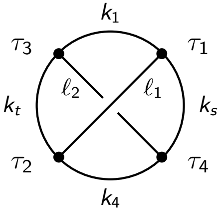
Interpretation:

- Semi-classical limit exhibits quasi-normal modes, JT $M(T_H)$ relation
- Interpret as black hole that emits and reabsorbs excitation

Result: OTO 4-point correlator

Schwarzian diagram:

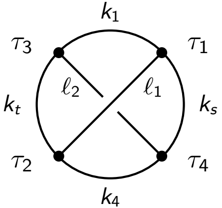
$$\left\langle \mathcal{O}_{\ell_1}(\tau_1, \tau_2)\mathcal{O}_{\ell_2}(\tau_3, \tau_4) \right\rangle_{\text{OTO}} =$$



Result: OTO 4-point correlator

Schwarzian diagram:

$$\left\langle \mathcal{O}_{\ell_1}(\tau_1,\tau_2)\mathcal{O}_{\ell_2}(\tau_3,\tau_4)\right\rangle_{\text{OTO}} =$$



$$= \int d\mu(k_i) e^{-k_1^2(\tau_3-\tau_1)-k_t^2(\tau_3-\tau_2)-k_4^2(\tau_4-\tau_2)-k_s^2(\beta-\tau_4+\tau_1)}$$

$$\times \gamma_{\ell_1}(k_1,k_s)\gamma_{\ell_2}(k_s,k_4)\gamma_{\ell_1}(k_4,k_t)\gamma_{\ell_2}(k_t,k_1) \times \begin{Bmatrix} \ell_1 & k_1 & k_s \\ \ell_2 & k_4 & k_t \end{Bmatrix}$$

- $6j$-symbol of $\mathrm{SL}(2,\mathbb{R})$ represents crossing of lines (OTO)
- Descends from R-matrix in 2d CFT

Application: Shockwaves from semiclassics

Semiclassical regime: $C \to +\infty$

$$\int_0^{+\infty} dq_+ \int_0^{+\infty} dp_- \Psi_1^*(q_+)\Phi_3^*(p_-)\mathcal{S}(p_-, q_+)\Psi_2(q_+)\Phi_4(p_-)$$

Application: Shockwaves from semiclassics

Semiclassical regime: $C \to +\infty$

$$\int_0^{+\infty} dq_+ \int_0^{+\infty} dp_- \Psi_1^*(q_+)\Phi_3^*(p_-)\mathcal{S}(p_-, q_+)\Psi_2(q_+)\Phi_4(p_-)$$

- Ψ, Φ = Kruskal wavefunctions = bulk-to-boundary propagators
- $\mathcal{S} = \exp\left(\frac{i\beta}{4\pi C}\, p_- q_+\right)$ the Dray-'t Hooft shockwave $\mathcal{S}$-matrix

Application: Shockwaves from semiclassics

Semiclassical regime: $C \to +\infty$

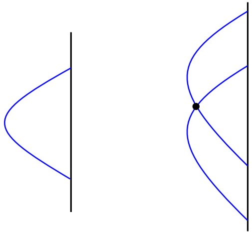
$$\int_0^{+\infty} dq_+ \int_0^{+\infty} dp_- \Psi_1^*(q_+)\Phi_3^*(p_-)\mathcal{S}(p_-, q_+)\Psi_2(q_+)\Phi_4(p_-)$$

- $\Psi, \Phi =$ Kruskal wavefunctions $=$ bulk-to-boundary propagators
- $\mathcal{S} = \exp\left(\frac{i\beta}{4\pi C}\, p_- q_+\right)$ the Dray-'t Hooft shockwave $\mathcal{S}$-matrix

Precise match with known semiclassics Shenker-Stanford '15

Alternative perspective: Wilson lines

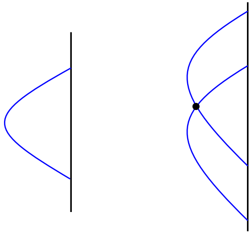
Alternatively, without resorting to 2d Liouville CFT
Bulk perspective on bilocal operators as boundary-anchored Wilson lines in $\mathrm{SL}(2,\mathbb{R})$ BF theory



Crossing of Wilson lines $\Rightarrow$ $6j$-symbol of group

Alternative perspective: Wilson lines

Alternatively, without resorting to 2d Liouville CFT
Bulk perspective on bilocal operators as boundary-anchored Wilson lines in $SL(2, \mathbb{R})$ BF theory



Crossing of Wilson lines $\Rightarrow$ $6j$-symbol of group

Thank you!