F is for Fiber:
Theories of Class F and Their Anomalies

Craig Lawrie

1806.06066 with Dario Martelli and Sakura Schäfer-Nameki

Class S [Gaiotto '09] : M5-branes on $\boxed{M_6 = M_4 \times C_{g,n}}$

→ 4d $\mathcal{N} = 2$ field theory

→ gauge couplings ↔ moduli of $C_{g,n}$

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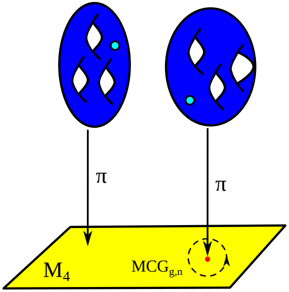
Class F: M5-branes on $\boxed{C_{g,n} \hookrightarrow M_6 \xrightarrow{\pi} M_4}$

→ fibration by punctured Riemann surfaces

→ couplings vary on spacetime

→ singular fibers/duality defects



6d $(2,0)$ SCFT of type G has anomaly polynomial

$$I_8 = \frac{r_G}{48}\left[p_2(N_5) - p_2(M_6) + \frac{1}{4}(p_1(N_5) - p_1(M_6))^2\right] + \frac{h_G^\vee d_G}{24} p_2(N_5)$$

Integration over the fiber $\rightsquigarrow$ pushforward

$$I_6 \supseteq \int_{\text{fiber}} I_8 = \pi_* I_8$$

For $C = T^2$

$$\pi_* I_8 = I_6 = \frac{1}{2} d_G c_3(\mathcal{S}_6^+) - \frac{1}{2} r_G c_2(\mathcal{S}_6^+) c_1(\mathcal{L}_D) - \frac{1}{24}(-6 r_G) c_1(\mathcal{L}_D) p_1(M_4) - \frac{61}{4} r_G c_1(\mathcal{L}_D)^3 + r_G \left[\frac{\mathfrak{a}_2}{4} c_1(\mathcal{L}_D)^2 S_{G_F} - \frac{\mathfrak{a}_1}{8} c_1(\mathcal{L}_D) S_{G_F}^2 + \frac{\mathfrak{a}_0}{4} S_{G_F}^3 \right]$$

$\mathcal{L}_D \to M_4$ is line bundle with connection $Q = -\frac{1}{2\mathrm{Im}(\tau)} d\mathrm{Re}(\tau)$

S_{G_F} 2-form related to singular fiber/defect flavour symmetry G_F

Class F with $C = T^2$ is $\underline{\mathcal{N} = 4 \text{ SYM with duality defects}}$

$\Rightarrow$ alternate 4d computation of I_6

In background $\gamma \in SL(2,\mathbb{Z})$ requires compensating $U(1)_D$ chiral rotation

[Bachas, Bain, Green; Martucci; Assel, Schäfer-Nameki]

$$\gamma : \tau \to \frac{a\tau + b}{c\tau + d}, \quad \begin{pmatrix} a & b \\ c & d \end{pmatrix} \in SL(2,\mathbb{Z}) \quad \to \quad e^{i\alpha(\gamma)} \equiv \frac{c\tau + d}{|c\tau + d|} \in U(1)_D$$

Gauginos charged $\pm 1/2$ under $U(1)_D \Rightarrow \underline{U(1)_D \text{ anomaly}}$

$$I_6 = \frac{1}{2} d_G\, c_3(\mathcal{S}_6^+) - \frac{1}{2} r_G\, c_2(\mathcal{S}_6^+) c_1(\mathcal{L}_D) - \frac{1}{24}(2r_G)\, c_1(\mathcal{L}_D) p_1(M_4) - \frac{1}{12} r_G\, c_1(\mathcal{L}_D)^3$$

Difference = 6d approach sensitive to duality defects

Universal contribution to anomaly from defects

$$I_6^{\text{uni,defects}} = -\frac{1}{24}(8r_G)c_1(\mathcal{L}_D)p_1(M_4)$$

Crosscheck: class F with $C = T^2$ on $\mathbb{R}^{1,1} \times \Sigma \rightsquigarrow$ 2d $(0,4)$ SCFTs

- "Strings" in 6d: $c_R = 3d_G\,\Sigma \cdot \Sigma + 3r_G\,c_1(\mathcal{L}_D) \cdot \Sigma$ [Vafa; Berman, Harvey; Haghighat, Murthy, Vafa, Vandoren; Shimizu, Tachikawa; CL, Schäfer-Nameki, Weigand]
- Holography: $\mathrm{AdS}_3 \times S^3/\Gamma \times (T^2 \hookrightarrow CY_3 \to B_2)$ [Couzens, CL, Martelli, Schäfer-Nameki, Wong]

If your appetite has been whetted...

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Theories of Class F and Their Anomalies

Craig Lawrie (Heidelberg), Dario Martelli (KCL), Sakura Schäfer-Nameki (Oxford)

Theories of Class F

Let $C_{g,n}$= genus g curve, n marked points. Then [Gaiotto]:

Class S: M5s on $\boxed{M_6 = C_{g,n} \times M_4}$ $\longrightarrow$ 4d $\mathcal{N}=2$ Theories

Goal: Generalise the 4d $\mathcal{N}=2$ class S theories to include

- Spacetime dependent gauge couplings ⇒ $C_{g,n}$-fibration
- Duality defects ⇒ Singularities of fibration

Class F: M5s on $\boxed{\begin{array}{ccc} C_{g,n} & \hookrightarrow & M_6 \\ & & \downarrow \\ & & M_4 \end{array}}$ $\longrightarrow$ 4d $\mathcal{N}=2$ with varying couplings and duality defects.

Simplest case: $C_{1,0}=T^2$ results in 4d $\mathcal{N}=4$ SYM with duality defects

Anomaly Polynomials of Class F

The 6d $\mathcal{N}=(2,0)$ SCFT of type G has an anomaly polynomial

$$I_8 = \frac{d_G h_G^\vee}{24} p_2(N_5) + \frac{r_G}{48}\left[p_2(N_5) - p_2(M_6) + \frac{1}{4}(p_1(N_5) - p_1(M_6))^2\right].$$

Class F: 'pushforward' I_8 along the $C_{g,n}$ fiber

$$\boxed{I_6 \supseteq \int_{\text{fiber}} I_8 = \pi_* I_8}$$

$C = T^2$: Fibration data contains $\mathcal{L}_D \to M_4$ "Weierstrass line bundle"

- 'Bulk' contribution from pushforward: [S_6= Spin bundle of $SU(4)_R$]

$$I_6 = \frac{d_G}{2} c_3(S_6) - \frac{r_G}{2} c_2(S_6) c_1(\mathcal{L}_D) + \frac{r_G}{4} p_1(M_4) c_1(\mathcal{L}_D) - \frac{61}{4} r_G c_1(\mathcal{L}_D)^3$$

- Duality defect contributions: universal

$$\begin{aligned} I_6^{\text{defects,universal}} &= \frac{1}{24} 8 r_G\, c_1(\mathcal{L}_D) p_1(M_4) \\ I_6^{\text{defects, non-uni}} &= \frac{\mathfrak{a}_2}{4} c_1(\mathcal{L}_D)^2 S_{G_F} - \frac{\mathfrak{a}_1}{8} c_1(\mathcal{L}_D)\, S_{G_F}^2 + \frac{\mathfrak{a}_0}{4} S_{G_F}^3 . \end{aligned} \tag{1}$$

$\mathfrak{a}_i$ depend on Kodaira singular fibers over $S_G \subset M_4$.

$C_{g,n}$: Pushforward for plane-curve fibrations gives $c_1(\mathcal{L}_D)$-corrections to class S anomaly polynomial, see [1].

Check 2: Anomalies of Strings

Class F brane realization: D3s in F-theory on elliptic $\mathrm{CY}_3 \to B$:

$\boxed{\text{Wrapped D3-branes } \Sigma \subset B \ \equiv\ \text{Class F on } M_4 = \Sigma \times \mathbb{R}^{1,1}}$

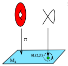
$$\int_\Sigma I_6^{\text{class F}} = d_G\,(c_2(F) - c_2(R))\,\Sigma\cdot\Sigma - \frac{1}{2} r_G \left(c_2(R) + c_2(F) - \frac{1}{2} p_1(M_2)\right) c_1(B_2)\cdot\Sigma$$

- Matches anomaly of $(0,4)$ SCFTs, $c_R = 3 d_G \Sigma\cdot\Sigma + 3 r_G c_1(B_2)\cdot\Sigma$
- Matches holographic computation in dual AdS_3 F-theory solution [2]

$$\begin{array}{c} T^2 \hookrightarrow \mathrm{CY}_3 \\ \downarrow \\ \mathrm{AdS}_3 \times S^3/\Gamma \times B_2 \end{array}$$

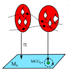
Duality Defects

Class F has real codimension 2 defects, around which the gauge couplings undergo duality transformations.



Class F for $C = T^2$

- T^2-fibration
  ⇒ $\tau = \frac{\theta}{2\pi} + i\frac{4\pi}{g^2}$ varies over M_4
- Singular T^2-fibers
  ⇒ $SL(2,\mathbb{Z})$ duality defects
- Flavor symmetry
  ⇒ Singular fiber type [Kodaira]



Class F for $C_{g,n}$:

- $C_{g,n}$-fibration
  ⇒ gauge couplings vary on M_4
- n-marked points = n sections
- Singular fibers
  ⇒ Mapping class group $\mathrm{MCG}_{g,n}$ duality defects

What can be computed? Anomaly polynomials.

Check 1: $\mathcal{N}=4$ SYM w/ Duality Defects

For $C = T^2$ class F, there is a 4d derivation of anomaly polynomial:

- $\gamma \in SL(2,\mathbb{Z})$ of 4d $\mathcal{N}=4$ induces $U(1)_D$ rotation of fermions/supercharges

$$\gamma: \quad \tau \to \frac{a\tau+b}{c\tau+d} \quad \to \quad e^{i\alpha(\gamma)} \equiv \frac{c\tau+d}{|c\tau+d|} \in U(1)_D .$$

  Chiral fermions are sections of $U(1)_D$ line bundle $\mathcal{L}_D \to M_4$
  ⇒ $U(1)_D$ anomaly
- Anomaly polynomial for 4d $\mathcal{N}=4$ SYM has extra terms:

$$\boxed{I_6 = \frac{d_G}{2} c_3(S_6) - \frac{r_G}{2} c_2(S_6) c_1(\mathcal{L}_D) - \frac{r_G}{12} p_1(M_4) c_1(\mathcal{L}_D) + \cdots}$$

  $\cdots$ = the non-universal $c_1(\mathcal{L}_D)$ terms I_6^{defects} (1).
- $c_1(\mathcal{L}_D) = \mathrm{d}Q$, where $Q = -\frac{1}{2\mathrm{Im}(\tau)} \mathrm{dRe}(\tau)$, contributes when τ is spacetime dependent, i.e. in class F for $C = T^2$

Outlook

- Include punctures into I_8 and pushforward to class F.
- Other theories, e.g. 6d $\mathcal{N}=(1,0)$ SCFTs, on $C_{g,n}$ fibrations.
- Microscopic description of duality defects.

Based on:

[1] C. Lawrie, D. Martelli and S. Schafer-Nameki, *Theories of Class F and Anomalies*, 1806.06066.

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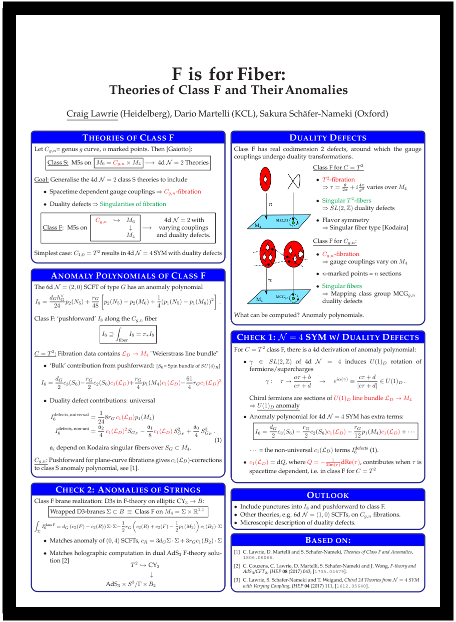

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