Membrane dynamics and Gravity

Based on arXiv:1712.09400

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Large D membrane paradigm [2015 - ongoing]

- Main point: Dynamics of black hole $\rightarrow$ Dynamics of membrane
  codimension-1 timelike non-gravitational hypersurface
  embedded in unperturbed (e.g. flat) background spacetime.
- Membrane variables: shape and velocity field u^μ $(D-1)$
- Membrane equations obtained in a perturbation series in $1/D$.
- Membrane localized Stress tensor and Entropy current

$$16\pi T_{\mu\nu} = \left(K\ \mathcal{P}_{\mu\nu} - 2\sigma_{\mu\nu}\right) + \left(K_{\mu\nu} - K g_{\mu\nu}\right) + (\dots)$$

$$J^\mu_S = \frac{u^\mu}{4} + (\dots) \qquad \Longrightarrow \hat{\nabla}_\mu J^\mu_S = \frac{1}{2K}\sigma_{\alpha\beta}\sigma^{\alpha\beta} + (\dots)$$

- (D) Spacetime Stress tensor conservation:
  $(D-1)$: $\hat{\nabla}^\mu T_{\mu\nu} = 0 \implies$ membrane equations at Large D
  (1): $K^{\mu\nu}T_{\mu\nu} = 0$ identically order by order
  $\implies$ Formulation consistent strictly at large D.
- Local form of 2nd law at large D

'Improved' arbitrary D membrane [arXiv:1712.09400]

- 'Improved' membrane Stress tensor and Entropy current

$$16\pi T_{\mu\nu} = \left(W\ \mathcal{P}_{\mu\nu} - 2\sigma_{\mu\nu}\right) + \left(K_{\mu\nu} - K g_{\mu\nu}\right) \qquad J^{\mu}_{S} = \frac{u^{\mu}}{4}$$

$$W \equiv \frac{K^2 - K^{\mu\nu}K_{\mu\nu} + 2K^{\mu\nu}\sigma_{\mu\nu}}{K + u.K.u} \qquad \hat{\nabla}_{\mu} J^{\mu}_{S} = \frac{1}{2W}\sigma_{\alpha\beta}\sigma^{\alpha\beta}$$

in any background that solves Einstein equations.

- $K^{\mu\nu}T_{\mu\nu} = 0$ identically $\implies$ Formulation consistent for arbitrary D
- Exactly obeys local form of 2nd law
- Reduces to previous $T_{\mu\nu},\ J^{\mu}_{S}$ at leading order in large D
  $\implies$ Reproduces black hole physics "at least" at large D.
  $\implies$ So, a consistent finite D completion

- Note: $\rho = 0, \quad p = \frac{W}{16\pi}, \quad T = \frac{W}{4\pi}, \quad s = \frac{1}{4}, \quad \eta = \frac{1}{16\pi}, \quad \frac{\eta}{s} = \frac{1}{4\pi}$

Stationary solutions

- $u^\mu = \frac{k^\mu}{\sqrt{-k.k}}$ for a Killing vector k^μ. Also, $\sigma_{\alpha\beta} = 0$, $\hat{\nabla}.u = 0$.
- $\hat{\nabla}^\mu T_{\mu\nu} \implies \frac{K^2 - K_{\mu\nu}K^{\mu\nu}}{K + u.K.u} = \frac{4\pi T_0}{\sqrt{-k.k}}$: Shape equation
- Consider action:

$$S = \frac{1}{16\pi}\left[\frac{2}{D-2}\Lambda \int_V \sqrt{-G} + \int_M \sqrt{-g}\ \left(K - \frac{4\pi T_0}{\sqrt{-k.k}}\right)\right]$$

- $\delta S = 0$ under change in shape $\implies$ Shape equation
- Stationary stress tensor $T_{\mu\nu} = -\frac{2}{\sqrt{-g}}\frac{\delta S}{\delta g^{\mu\nu}}$
- Action onshell reduces to

$$S = \frac{E}{T_0} - S_{ent} = -\ln Z$$

- Note $\partial_\beta S = E$ onshell

Surprises at finite D

- Energy, Entropy and Temperature of static spherical membrane in AdS/Flat/dS matches at finite D with Schwarzschild black hole.

- Consider slowly varying planar membrane in AdS. Calculate the linearized gravitational field sourced by $T_{\mu\nu}$. Find the boundary stress tensor. After field redefinitions

$$\mathbb{T}_{\mu\nu} = p\left(\eta_{\mu\nu} + d\ \mathsf{v}_\mu \mathsf{v}_\nu\right) - 2\eta\sigma_{\mu\nu} + 2\eta\left(\frac{d}{4\pi\mathsf{T}}\right)\left(\sigma\sigma + \omega\omega + \omega\sigma + \mathsf{v}.D\sigma\right)$$

$$\text{where} \quad p = \frac{1}{16\pi}\left(\frac{4\pi\mathsf{T}}{d}\right)^{d}, \eta = \frac{1}{16\pi}\left(\frac{4\pi\mathsf{T}}{d}\right)^{d-1}, \quad (d = D-1)$$

- Matches with Fluid-Gravity upto first order in derivative expansion, even at finite d!
- Agrees with Fluid-Gravity at second order only in large d limit, with subleading deviations.

Summary

- Special kind membrane dynamics, well defined at arbitrary D.
- Simple shape equation and Action for stationary membranes. Action onshell reduces to thermal partition function.
- Reproduces black hole physics better than expected for finite D!

- Can simulations capture key features of black hole dynamics (e.g. mergers, Gregory-Laflamme instabilities)?
- Can we construct a membrane formulation such that it can reproduce all the stationary black hole solutions exactly?
- Can we understand dynamical Second law of black holes thermodynamics for higher derivative theories of gravity?

Thank you!

I invite you to see my poster