

M-theory S-Matrix from 6d CFT

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M-theory S-Matrix

- M-theory is a quantum theory of interacting supergravitons in 11d with no dimensionless coupling.
- Graviton S-matrix in small momentum ($\ell_{11} \ll 1$) expansion:

$$\begin{aligned}
 & \text{Contact Diagram} = \mathcal{A}(s, t, u) \\
 & = \text{Tree Exchange} + \sum_{m=0}^{\infty} \text{Tree Exchange} + \text{Loops} \\
 & = \ell_{11}^9 \mathcal{A}_R(s, t, u) + \ell_{11}^{15+2m} \mathcal{A}_{D^{2m}R^4}(s, t, u) + \dots
 \end{aligned}$$

- Protected terms from type IIA string theory + duality [Green, Tseytlin]:
 $\mathcal{A}_{R^4} = \mathcal{A}_R \frac{stu}{3 \cdot 2^7}$, $\mathcal{A}_{D^2 R^4} = \mathcal{A}_{D^4 R^4} = 0$, $\mathcal{A}_{D^6 R^4} = \mathcal{A}_R \frac{(stu)^2}{15 \cdot 2^{15}}$.
- Goal: Find all tree level terms $\mathcal{A}_{D^{2m}R^4}$ for $m > 3$ using AdS/CFT.

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 & \text{Four-point vertex} = \mathcal{A}(s, t, u) \\
 & = \text{Tree-level exchange} + \sum_{m=0}^{\infty} \text{Higher-order exchange} + \text{Loops} \\
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AdS/CFT for M-theory

- M-theory contains two non-perturbative dynamical objects: M2 branes and M5 branes.
- We study $(2, 0)$ A_{N-1} 6d SCFT that describes stack of N M5 branes, and is dual at large N to M-theory on $AdS_7 \times S^4$.
- We compute: $\mathcal{G}_k(U, V) = x_{12}^{2k} x_{34}^{2k} \langle \mathcal{O}_k^1 \mathcal{O}_k^2 \mathcal{O}_k^3 \mathcal{O}_k^4 \rangle$ of k -th lowest dimension half-BPS operators in CFT_6 in large N expansion.
 - $U \equiv \frac{x_{12}^2 x_{34}^2}{x_{13}^2 x_{24}^2}$, $V \equiv \frac{x_{14}^2 x_{23}^2}{x_{13}^2 x_{24}^2}$ are conformal cross ratios.
 - \mathcal{G}_k dual to correlator of k -th lowest KK modes of M-theory on $AdS_7 \times S^4$.
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Tree Level Half-BPS Four Point functions: Constraints

- We use Mellin transform $\mathcal{G}_k(U, V) \rightarrow M_k(s, t, u)$ [Mack, Penedones], where s, t, u are like Mandelstam variables in Mellin space.
- For N large, let $M_k^{\text{tree}}(s, t, u) \equiv \sum_{p=1}^{\infty} M_k^{(p)}(s, t, u)$, where p is degree in $s, t, u \rightarrow \infty$, and $M_k^{(p)}$ fixed in terms of CFT_6 data by:
 - 1 Crossing symmetry.
 - 2 Superconformal Ward identities [Dolan, Gallot, Sokatchev; Rastelli, Zhou].
 - 3 Poles in s, t, u correspond to dimensions of operators in $\mathcal{O}_k \times \mathcal{O}_k$, for **tree level** only allow poles for half-BPS operators.
- Flat space limit of $M_k^{(p)}(s, t, u)$ gives $2p$ derivative contribution to $\mathcal{A}|_{7d}(s, t, u)$ [Penedones], e.g. $M_k^{(1)} \rightarrow \mathcal{A}_R|_{7d}$.

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- $M_k^{(1)}$ fixed by central charge $\frac{1}{c_T} \approx N^{-3}$ [Rastelli, Zhou] $\Rightarrow \mathcal{A}_R|_{7d}$ is proportional to gravitational coupling $\kappa^2 \approx N^{-3}$ as expected.
- No $M_k^{(\rho)}$ for $\rho = 2, 3 \Rightarrow$ no $\mathcal{A}_{R^2}|_{7d}$ or $\mathcal{A}_{R^3}|_{7d}$ [SMC, Perlmutter].
- $M_k^{(\rho)}$ for $4 \leq \rho < 10$, which gives $\mathcal{A}_{D^{2\rho-8}R^4}|_{7d}$ in flat space limit, fixed by small set of CFT₆ OPE coefficients [SMC, Perlmutter].
 - $M_k^{(\rho)}$ for $\rho \geq 10$ has same N scaling as loop terms, so require loop Mellin amplitudes to fix unambiguously.
- $M_k^{(4)}$ fixed by half-BPS OPE coefficient λ_{BPS}^2 [SMC, Perlmutter] that can be computed exactly [Beem, Rastelli, van Rees].
 - Flat space limit of $M_k^{(4)}$ correctly reproduces the known $\mathcal{A}_{R^4}|_{7d}$!

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Conclusion

Results:

- Tree level $D^{2m}R^4$ contributions to 11d M-theory S-matrix for $m < 6$ in terms of CFT_6 data.
- Known half-BPS CFT_6 data precisely reproduces R^4 contribution.

Future Directions:

- Derive loop Mellin amplitudes \Rightarrow loop 11d S-matrix terms.
- 6d numerical bootstrap [Beem, Lemos, Rastelli, van Rees] to fix CFT_6 data \Rightarrow 11d S-matrix coefficients.
- Apply method to $\text{AdS}_{d+1}/\text{CFT}_d$ for other d .
 - See Silviu Pufu's talk tomorrow for $d = 3$ case [SCM, Pufu, Yin].

See my poster for more details!