SYK model with $\mathcal{N} = 2$ supersymmetry

Ksenia Bulycheva

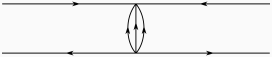
June 23, 2018

Princeton University

Generalizing SYK Model to two dimensions

- We need fields of dimension zero

$$\int dt \Psi D \Psi \longrightarrow \int d^2 z D\Phi \bar{D}\Phi. \tag{1}$$



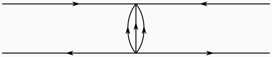
Generalizing SYK Model to two dimensions

- We need fields of dimension zero

$$\int dt \Psi D \Psi \longrightarrow \int d^2 z D\Phi \bar{D}\Phi. \tag{1}$$

- We need positive–definite potential

$$\int d^2 z J_{i_1 \dots i_q} \Phi_{i_1} \cdots \Phi_{i_q} \longrightarrow \int d^2 z \left(C_{i_1 \dots i_q} \Phi_{i_2} \cdots \Phi_{i_q} \right)^2 . \tag{2}$$



Generalizing SYK Model to two dimensions

- We need fields of dimension zero

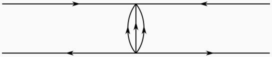
$$\int dt \Psi D \Psi \longrightarrow \int d^2 z D \Phi \bar{D} \Phi. \tag{1}$$

- We need positive–definite potential

$$\int d^2 z J_{i_1 \ldots i_q} \Phi_{i_1} \cdots \Phi_{i_q} \longrightarrow \int d^2 z \left(C_{i_1 \ldots i_q} \Phi_{i_2} \cdots \Phi_{i_q} \right)^2 . \tag{2}$$

- $\mathcal{N} = 2$ theory with holomorphic and homogeneous superpotential flows to a true conformal point

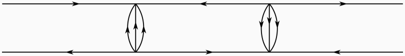
$$\mathcal{W} = C_{i_1 \ldots i_q} \Phi_{i1} \cdots \Phi_{i_q} . \tag{3}$$



One-dimensional superspace

- Supercoordinates

$$\mathbf{1} \equiv \left(\tau_1, \theta_1, \bar{\theta}_1\right), \qquad D_1 \equiv \frac{\partial}{\partial \theta_1} + \bar{\theta}_1 \frac{\partial}{\partial \tau}. \tag{4}$$



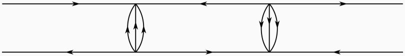
One-dimensional superspace

- Supercoordinates

$$\mathbf{1} \equiv \left(\tau_1, \theta_1, \bar{\theta}_1\right), \qquad D_1 \equiv \frac{\partial}{\partial \theta_1} + \bar{\theta}_1 \frac{\partial}{\partial \tau}. \tag{4}$$

- Chiral fields

$$D\bar{\Psi} = \bar{D}\Psi = 0. \tag{5}$$



One-dimensional superspace

- Supercoordinates

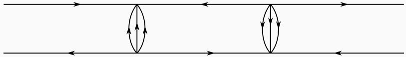
$$\mathbf{1} \equiv \left(\tau_1, \theta_1, \bar{\theta}_1\right), \qquad D_1 \equiv \frac{\partial}{\partial \theta_1} + \bar{\theta}_1 \frac{\partial}{\partial \tau}. \tag{4}$$

- Chiral fields

$$D\bar{\Psi} = \bar{D}\Psi = 0. \tag{5}$$

- Chirality and $SU(1,1|1)$ group $\Rightarrow$ bosonic cross-ratio

$$\chi = \frac{\langle 12 \rangle \langle 34 \rangle}{\langle 14 \rangle \langle 32 \rangle}, \qquad D_1 \langle 12 \rangle = \bar{D}_2 \langle 12 \rangle = 0. \tag{6}$$



Four-point function

Four-point function is a sum:

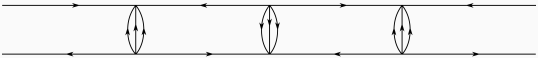
$$\mathcal{F} = \frac{\mathcal{F}_0}{1-K} = \sum_h \frac{1}{1-k(h)} \frac{\langle \mathcal{F}_0, \xi_h \rangle}{\langle \xi_h, \xi_h \rangle} \xi_h(\chi)\,. \tag{7}$$

$\xi_h(\chi)$ is an eigenfunction of the two–point Casimir:

$$\mathcal{C}(\chi)\,\xi_h(\chi) = h^2 \xi_h(\chi)\,. \tag{8}$$

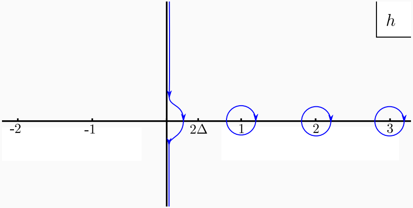
ξ_h is a linear combination of $\mathcal{N}=0$ eigenfunctions:

$$\xi_h = h\left(\Psi_h^A - \Psi_{-h}^S\right). \tag{9}$$



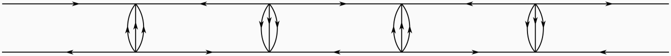
Integration contour

Normalizable states: $h \in i\mathbb{R}$ or $h \in \mathbb{Z}$.



$$\mathcal{F}(\chi) = -\alpha \sum_m \operatorname*{Res}_{h=h_m>0} \frac{1}{4\pi h \tan \pi h} \frac{1}{1-k(h)} \xi_h(\chi), \qquad k(h_m) = 1. \tag{10}$$

$$h = 1: \qquad \mathcal{Q} = R + \theta \bar{Q} + \bar{\theta} Q + \theta \bar{\theta} T. \tag{11}$$



Two dimensions

Superalgebra: $su(1,1|1) \oplus su(1,1|1)$

Normalizable states: $h = \frac{l}{2} + is, \qquad \tilde{h} = -\frac{l}{2} + is, \quad s \in \mathbb{R}, \quad l \in \mathbb{Z}.$

Four-point function:

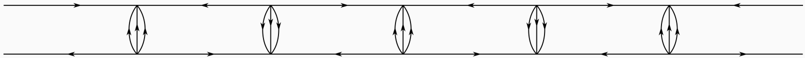
$$\mathcal{F}(\chi,\bar{\chi}) = \frac{1}{4\pi}\frac{\Delta}{1-\Delta}\sum_{l\in\mathbb{Z}}\int_{-\infty}^{\infty}\frac{ds}{2\pi}\frac{k\left(h,\tilde{h}\right)}{1-k\left(h,\tilde{h}\right)}\frac{\sin\pi h}{\cos\pi\tilde{h}}\varphi_h(\chi)\,\varphi_{\tilde{h}}(\bar{\chi})\,. \tag{12}$$

Central charge:

$$c = 3N(1-2\Delta)\,. \tag{13}$$

Maximal Lyapunov exponent $h+\tilde{h} \sim 0.6$:

$$f_R \sim e^{\left(h-\tilde{h}\right)x}e^{-\left(h+\tilde{h}\right)t}. \tag{14}$$



Thank you!