Developments in String Field Theory

Ashoke Sen

Harish-Chandra Research Institute, Allahabad, India

Okinawa, June 2018

What is string field theory?

String field theory is a quantum field theory with infinite number of fields in which perturbative amplitudes are computed by summing over Feynman diagrams.

Each Feynman diagram can be represented as an integral over the moduli space of a Riemann surface with

– the correct integrand (as in world-sheet description)

– but only a limited range of integration.

Sum over all Feynman diagrams reproduces the integration over the whole moduli space.

Why string field theory?

1. Since it is a quantum field theory with action, the S-matrix can be represented as path integral over string fields

– could possibly give a non-perturbative formulation of string theory

(has not been successful so far)

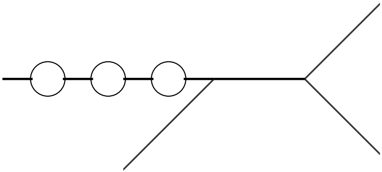
2. Classical string field theory may be useful for finding new classical solutions of string field theory that are not easily describable in conventional world-sheet formulation

(reasonably successful for open string field theory)

3. For a given amplitude, the usual <u>world-sheet</u> description of string perturbation theory gives one term at every loop order

– usually considered an advantage, but this may not always be the case.

Example: In a quantum field theory, self energy insertions on external legs have to be resummed separately to find the renormalized mass



requires

– separating graphs with self-energy insertions on external lines from other graphs

– off-shell 2-point function to look for renormalized pole position

String field theory is well suited for this task.

Therefore string field theory is needed to fully define string perturbation theory

– by itself a strong motivation for the subject

PLAN

1. Formulation of string field theory (SFT)

2. Classical solutions in open string field theory

3. Background independence

4. Application to perturbation theory

We shall focus on covariant string field theory, and not discuss light-cone string field theory.

Formulation of covariant string field theory

Tree level bosonic SFT (open and closed)

Witten; Saadi, Zwiebach; Kugo, Kunitomo, Suehiro; Sonoda, Zwiebach; Zwiebach; $\cdots$

For open strings,

$\mathcal{H} \equiv$ Vector space of boundary vertex operators / states of matter + ghost world-sheet CFT

For closed strings,

$\mathcal{H} \equiv$ Vector space of vertex operators / states of matter + ghost world-sheet theory subject to additional constraints:

$$\mathbf{b_0^-}|\mathbf{A}\rangle = \mathbf{0}, \qquad \mathbf{L_0^-}|\mathbf{A}\rangle = \mathbf{0} \quad \text{for } \mathbf{A} \in \mathcal{H}$$

$$\mathbf{b_0^\pm} = \mathbf{b_0} \pm \bar{\mathbf{b}}_\mathbf{0}, \quad \mathbf{L_0^\pm} = \mathbf{L_0} \pm \bar{\mathbf{L}}_\mathbf{0}, \quad \mathbf{c_0^\pm} = (\mathbf{c_0} \pm \bar{\mathbf{c}}_\mathbf{0})/\mathbf{2}$$

String field Φ: An arbitrary vertex operator / state in $\mathcal{H}$ with

– ghost number 1 for open strings

– ghost number 2 for closed strings

Φ is

– grassmann odd for open strings

– grassmann even for closed strings

due to odd / even ghost number of Φ.

Action:

$$\textbf{Open}: \quad S = \frac{1}{2}\langle\Phi|Q|\Phi\rangle + \sum_n \frac{1}{n}\{\Phi^n\}$$

$$\textbf{Closed}: \quad S = \frac{1}{2}\langle\Phi|c_0^- Q|\Phi\rangle + \sum_n \frac{1}{n!}\{\Phi^n\}$$

Q: BRST operator in the matter-ghost CFT

For $A_1, \cdots A_n \in \mathcal{H}$, $\{A_1 \cdots A_n\}$ is an n-point correlation function of the vertex operators $A_1, \cdots A_n$ on the disk / sphere $\cdots$

$\cdots$ integrated over some specific codimension 0 subspace of the moduli space of disk / sphere with n punctures.

The vertex operators are inserted on the disk / sphere with some specified local coordinates around the punctures.

$\{\mathbf{A_1 \cdots A_n}\}$ **has**

– cyclic symmetry for open strings

– full permutation symmetry for closed strings

for grassmann odd / even $\mathbf{A_1, \cdots A_n}$

$[\mathbf{A_1 \cdots A_n}] \in \mathcal{H}$ **is defined via**

$$\langle \mathbf{C} | [\mathbf{A_1 \cdots A_n}] \rangle = \{\mathbf{C A_1 \cdots A_n}\} \quad \forall\, \mathbf{C} \in \mathcal{H} \qquad \textbf{for open strings}$$

$$\langle \mathbf{C} | \mathbf{c_0^-} | [\mathbf{A_1 \cdots A_n}] \rangle = \{\mathbf{C A_1 \cdots A_n}\} \quad \forall\, \mathbf{C} \in \mathcal{H} \qquad \textbf{for closed strings}$$

Infinitesimal gauge transformation:

$$\delta\Phi = \mathbf{Q}\Lambda + \sum_{\mathbf{m},\mathbf{n}} (-\mathbf{1})^{\mathbf{m}+\mathbf{1}} [\Phi^{\mathbf{m}} \Lambda \Phi^{\mathbf{n}}] \quad \textbf{for open strings}$$

$$\delta\Phi = \mathbf{Q}\Lambda + \sum_{\mathbf{n}} \frac{\mathbf{1}}{\mathbf{n}!} [\Lambda \Phi^{\mathbf{n}}] \quad \textbf{for closed strings}$$

Gauge transformation parameter Λ: a state in $\mathcal{H}$ with

– ghost number 0 for open strings

– ghost number 1 for closed strings

Gauge fixing: Introduce ghosts

Set of all ghost fields + matter fields: A state in $\mathcal{H}$ with arbitrary ghost number

Bochicchio; Thorn; Sonoda, Zwiebach

Action with matter + ghost has the same form as the original action, but there is no constraint on the ghost number of Φ

Natural framework: Batalin-Vilkovisky (BV) formalism

Each tree Feynman diagram gives integration over part of the moduli space of punctured disk / sphere.

Sum over all Feynman diagrams gives integration over the full moduli space.

Giddings; Giddings, Martinec, Witten; Saadi, Zwiebach

Loops:

Generically open string field theory requires coupling to closed string field theory

Zwiebach

Closed string field theory makes sense by itself

Action has similar form except that definition of $\{A_1 \cdots A_n\}$ requires additional contribution from integration over subspaces of moduli space of higher genus surfaces

– additional finite 'local' counterterms needed for gauge invariance

– also ensures that the sum over all Feynman diagrams gives correct result for on-shell amplitudes

Exception: Cubic open string field theory

Witten

– uses special choice of local coordinates for defining $\{A_1 A_2 A_3\}$

1. No vertices other than cubic are needed either at tree or at loop level.

2. No need to couple to closed strings (for on-shell amplitudes)

(closed string poles appear automatically from the UV region of open SFT)

Special notation: $[AB] = A * B$ or just AB

The SFT's constructed this way are not unique.

Different possible choice of local coordinate system at the punctures lead to different string field theories . . .

. . . **related by field redefinition** Hata, Zwiebach; $\cdots$

Nevertheless some choice may be simpler than the others

e.g. the cubic open SFT

There is no analogous 'simple choice' for closed SFT

A particular proposal involves using local coordinates induced by 'minimal area metric' Zwiebach

– minimizes the area of the Riemann surface subject to a lower bound on the length of all non-trivial closed geodesics

– hard to construct such metrics and very few explicit examples are known

Recent progress:

1. Moosavian and Pius proposed a choice of local coordinates based on constant negative curvature metric on the Riemann surface

– the metric needs to be modified near the boundaries of moduli spaces but systematic construction is possible

2. Headrick and Zwiebach developed systematic tools for (numerically) constructing the minimal area metric

Both approaches are useful for providing explicit definition of off-shell string amplitudes

Superstring field theory (in RNS formulation)

There are two equivalent approaches to perturbation theory

1. Integration over supermoduli space

2. Use of picture changing operators (PCO)

Friedan, Martinec, Shenker

So far mostly the second approach has been used in the construction of SFT.

The Hilbert space becomes a direct sum of Hilbert spaces of different picture numbers

$$\mathcal{H} = \oplus_{\mathbf{n}} \mathcal{H}_{\mathbf{n}}$$

n $\in \mathbb{Z}$: NS sector – all n are equivalent on-shell

n $\in \mathbb{Z} + \frac{1}{2}$: R-sector – all n are equivalent on-shell

Canonical choice of open string field: $\Phi \in \mathcal{H}_{-1} + \mathcal{H}_{-1/2}$

Canonical choice of closed string field:

$$\Phi \in \mathcal{H}_{-1,-1} + \mathcal{H}_{-1/2,-1} + \mathcal{H}_{-1,-1/2} + \mathcal{H}_{-1/2,-1/2}$$

For a g-loop amplitude of m NS and n R-sector states, we need to insert $(2g-2+m+n/2)$ PCO's for picture number conservation

Result is independent of the locations of the PCO's if we choose them avoiding 'spurious singularities'

– any singularity that depends on PCO locations e.g. those associated with PCO collisions

There is a systematic procedure for avoiding this

A.S.; A.S., Witten

However off-shell amplitudes depend on PCO locations just as they depend on the local coordinates at the punctures.

For NS sector fields (NSNS for closed strings) the tree level string field theory can be constructed following the same procedure as bosonic string field theory

Saroja, A.S.; Erler, Konopka, Sachs

– PCO locations enter in the definition of $\{A_1 \cdots A_n\}$.

Different string field theories associated with different choices of local coordinates and PCO locations are related by field redefinition.

Ramond sector has additional complications associated with kinetic term.

Two Φ's have total picture number -1 since $\Phi \in \mathcal{H}_{-1/2}$

We need an additional picture number -1 at genus 0

This is related to the absence of a covariant kinetic term for type IIB supergravity due to self-dual 5-form field strength.

Resolution: In the R sector introduce another string field $\Psi \in \mathcal{H}_{-3/2}$ and write the kinetic term as A.S.

$$-\frac{1}{2}\langle\Psi|Q\,G|\Psi\rangle + \langle\Psi|Q|\Phi\rangle$$

G: zero mode of PCO

The interaction terms involve the original fields Φ

Result: One combination of Ψ and Φ remains free field and decouples from the theory

The rest of the degrees of freedom describe correctly the interacting string field theory.

This construction works at the loop level exacty as in the case of bosonic string theory.

The string field theory action depends on the choice of local coordinates at the punctures and the choice of PCO locations.

All such theories are related to each other by field redefinition.

Question: Is there a 'simple choice' that is simpler than the others?

Difficult for closed string theory since even the bosonic closed string field theory does not have a simple choice.

However progress has been made in <u>tree</u> level open superstring field theory.

In 1995 Berkovits constructed a tree level open superstring field theory with string field Φ in the large Hilbert space

– contains states of arbitrary integer picture number.

1. Has many more fields but also extra gauge invariance that reduces the number of degrees of freedom.

2. The action can be written in a compact form

It is hard (but possible for simple cases) to show that the sum of Feynman diagrams gives the correct amplitude

Berkovits, Echevarria

In 2013, Erler, Konopka and Sachs constructed an open superstring field theory for NS sector fields generalizing an earlier construction of Witten. also Iimori, Noumi, Okawa, Torii

1. Local coordinates at the punctures are similar to those used in cubic open bosonic string field theory.

2. PCO insertions are smeared so as to alleviate the singularities arising from PCO collisions.

3. Need to add higher order vertices, but there are simple algorithms to generate them maintaining gauge invariance.

4. Reproduces correctly tree amplitudes of NS sector states. Konpka

Recently Berkovits open superstring field theory was shown to reduce to the formulation based on PCO insertions after partial gauge fixing

Iimori, Noumi, Okawa, Torii; Erler, Okawa, Takezaki; Erler

– also proves indirectly that Berkovits theory correctly reproduces the tree level NS sector amplitudes of open superstring field theory

Ramond sector:

The doubling trick can be used to get a simple version of open superstring field theory including Ramond sector fields — Erler, Okawa, Takezaki; Konopka, Sachs

1. Uses local coordinate systems similar to the one for cubic open bosonic SFT

2. Systematic construction of smeared PCO insertions

A different formulation of the Ramond sector was given by Kunitomo and Okawa.

Instead of taking $|\Phi\rangle \in \mathcal{H}_{-1/2}$, we put additional constraints on Φ.

– analog of the $L_0^-|\Phi\rangle = 0$ constraint for closed strings but more complicated.

1. The action is quadratic in the Ramond sector fields but has higher powers of NS sector fields.

2. Checking that Feynman diagrams reproduce the amplitudes is hard

– tested up to some tree level 5-point functions

Recently a new approach to classical open string field theory has been developed Ohmori, Okawa

– based on integration over supermoduli space

– still in its infancy

Classical solution in open string field theory

1. **Numerical (level truncation)**

Kostelecky, Samuel; Kostelecky, Potting; A.S., Zwiebach; Moeller, Taylor; Gaiotto, Rastelli; $\cdots$

2. **Analytical**

Schnabl, $\cdots$

Numerical solution on D-branes

1. Expand the string field in a basis of L_0 eigenstates

$$\Phi = \sum_n a_n |\phi_n\rangle$$

2. Truncate the expansion to those $|\phi_n\rangle$ with L_0 eigenvalue below some fixed number L, and satisfying appropriate symmetry restrictions

3. Evaluate the action, and extremize with respect to the a_n's to find solution.

4. Hope that as we increase L, the solution converges.

This has successfully generated many solutions in Witten's cubic bosonic open string field theory and Berkovits' open superstring field theory.

Examples:

1. Tachyon vacuum describing vacuum without D-branes

2. Solitons describing lower dimensional D-branes

The energies carried by these solutions come quite close to the expected energies of the D-brane configurations they attempt to describe.

Can we use this approach to construct new solutions that are not known otherwise?

– new boundary conformal field theories.

Recently some solutions have been found with no obvious description as D-branes.

Kudrna, Schnabl, Vosmera, to appear

Set-up: D2 brane of bosonic string theory wrapped on T^2

$\exists$ numerical solution which do not have obvious CFT interpretation except at special points in the moduli space of T^2.

At special point in the moduli space one can construct boundary state analytically with the help of enhanced chiral algebra.

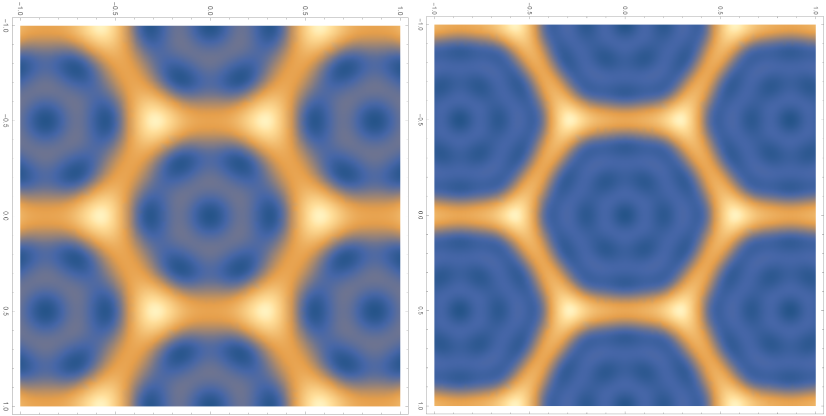
Kudrna, Schnabl, Vosmera; also earlier work by Yi, Kane; Affleck, Oshikawa, Saleur

$B = 0, \quad R_1 = R_2 = \sqrt{2\alpha'}, \quad$ angle=$2\pi/3$

The energy density profile agrees between numerical and analytical results.

Comparison between numerical and analytic results for energy density profile

Kudrna, Schnabl, Vosmera, to appear



– demonstrates the utility of open string field theory in looking for new conformally invariant boundary conditions

Analytic solutions in cubic open string field theory

First solution due to Schnabl

– describes the tachyon vacuum on the Dp-brane

The energy density of the vacuum
= - the tension of the Dp-brane

$\Rightarrow$ described vacuum without any D-brane

Solution is given in ‘Schnabl gauge’ and not in Siegel gauge, and cannot be directly compared with earlier analysis based on level truncation

Subsequent developments have provided better understanding of this solution and also paved the way to finding more analytic solutions.

Okawa; Rastelli, Zwiebach, Erler, $\cdots$

Computation of Veneziano amplitude in the Schnabl gauge gives the correct result confirming validity of this gauge choice

Rastelli, Zwiebach

A rewriting of Schnabl's solution:

$$\Phi = e^{-K/2}\, c\, K\, B\, (1 - e^{-K})^{-1}\, c\, e^{-K/2}$$

K, B, c: String field constructed respectively from

– line integral of stress tensor

– line integral of b-ghost

– insertion of c-ghost

in specific coordinate system.

All products are $*$-products

Other analytic solutions

If the world-sheet theory has a boundary marginal deformations by some operator V, then we have a family of boundary CFT

– provides consistent background for open string theory

There must be a family of solutions in open string field theory describing the family of consistent backgound

Can we construct such solutions analytically?

Yes, if VV operator product has no singularity.

Schnabl; Kiermaier, Okawa, Rastelli, Zwiebach; $\cdots$

Absence of singularity in VV OPE gives a very limited choice, containing e^{aX^0} factors in V

Standard time independent operators will have $(z-w)^{-2}$ singularities.

Remedy: Add a term proportional to $i\partial X^0$ to cancel this pole in OPE

– corresponds to switching on a constant A_0 (pure gauge)

There have been many subsequent developments simplifying the construction that paved the route for further developments.

Fuchs, Kroyter, Potting; Kiermaier, Okawa

Question: Given a new solution, how do we know what it represents?

Answer: See how a closed string feels the solution

A systematic procedure for carrying this out was developed by Ellwood

Ellwood invariant: Senses the coupling of a closed string to the open string field

This was later used to give an algorithm for constructing a boundary state associated with a given solution of open string field theory. **Kiermaier, Okawa, Zwiebach; Kudrna, Maccaferri, Schnabl**

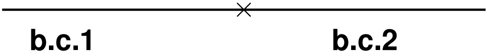
Can we construct analytic solutions associated with non-marginal deformations?

e.g. take a D1 brane wrapped on a circle of radius R . . .

. . . **and find a solution representing a D0 brane**

Strategy: Make use of boundary condition changing operators

Erler, Maccaferri, based on Kiermaier, Okawa, Soler



One can construct solutions in the string field theory constructed using boundary CFT with b.c.1 that describes boundary CFT with b.c.2

Solution takes a suggestive form:

$$\Phi = \Psi_{\mathbf{tv}} - \Psi_*$$

$\Psi_{\mathbf{tv}}$: tachyon vacuum solution in b.c.1

Ψ_*: tachyon vacuum solution in b.c.2, ‘written in the variables of the string field theory around b.c.1’

– as if we first create the tachyon vacuum in b.c.1 and then create the D-brane associated with b.c.2 from there

Classical solution in superstring field theory

Still in its infancy

Erler

An open problem:

Given a string field configuration that satisfies equations of motion, how do we know if the solution is singular or non-singular?

There is no natural norm in the space of string fields that can be used to demand normalizability.

There are currently many solutions, including strange ones with negative tension, whose interpretation is unclear due to this issue.

So far all solutions found in Siegel gauge level truncation seem to be sensible, but this is too restrictive.

Background independence

Construction of string field theory requires choosing a classical background

– corresponds to a choice of (boundary) CFT describing the world-sheet theory.

The action is then given in terms of the correlation functions and BRST operator in this world-sheet theory.

Question: How are string field theories constructed using two different CFT's related?

When the two CFT's are related by marginal deformation the answer is simple

The two string field theories are related by a field redefinition.

For bosonic string field theories the explicit field redefinition relating the two theories was constructed in the 90's **A.S., Zwiebach**

– constructed for infinitesimal deformation which could be integrated to give the result for finite deformation

This construction has now also been generalized to superstring field theories, including the Ramond sector **A.S.**

One subtle point: Only the interacting part is background independent.

The extra free string field action that one requires for the construction of the action is background dependent.

For open string field theory one can do better using boundary condition changing operators

– can actually construct explicit field redefinitions relating string field theories formulated around finitely separated backgrounds.

Erler, Maccaferri

Application to perturbation theory

Using the description of amplitudes as sum over Feynman diagrams, one can use techniques of quantum field theory to prove general properties of these amplitudes.

Rudra, Pius, A.S.; Pius, A.S.; A.S

1. **Unitarity of perturbative S-matrix**

2. **Systematic procedure for computing finite mass renormalization of massive particles**

3. **Systematic procedure for computing string amplitudes when the perturbative vacuum gets destabilized and settles down to a nearby stable vacuum**

Summary

We now have a complete formulation of field theories of all string theories, although the formulation of closed string field theories remain complicated.

The most ambitious goal of string field theory would be to give a non-perturbative definition of string theory

– has not been realized so far.

Nevertheless on two fronts it has had reasonable success:

1. Classical solutions in open string field theory

2. A systematic procedure for string perturbation theory in case of finite mass renormalization or shift in the vacuum.