Black holes as normal quantum systems

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Is a BH a normal quantum system?

- One can try to interpret the Euclidean path integral for a black hole as a partition function [Gibbons, Hawking] of a quantum system. One wants to find (say in global AdS)

$$\int \mathcal{D}g_{\mu\nu} e^{-I[g]} = \sum_n e^{-\beta E_n}. \tag{1}$$

  Semiclassically (small G_N), one finds $e^{-\beta E(\beta)+S(\beta)}$ with reasonable S, E for thermodynamics.
- However, in some cases the integral can be carried out exactly and leads to a bad (negative and/or continuous) spectrum e.g. [Maloney, Witten].
- So (1) can fail in a UV-compete gravity theory (but see [Dabholkar, Gomes, Murthy] for a more optimistic case).

Enter string theory

- In string theory the situation is much more promising, since BHs have been connected to string or brane configurations with comparable but "normal" entropy [Susskind, Sen, Horowitz/Polchinski, Strominger/Vafa, Maldacena].
- In string theory, it seems likely that BH are indeed normal quantum systems.

In fact, via AdS/CFT, this perspective has taught us a lot. BHs give large N strongly interacting models of:

- thermalization
- transport and hydrodynamics
- entanglement dynamics
- aspects of quantum information
- ...

It has brought string theorists together with condensed matter and quantum information people.

A hope for the future

- However, we still want to be able to find the discreteness of energy levels from the BH perspective.
- It isn't reasonable to hope to actually calculate the energy levels. Proving they are discrete may also be unreasonable. But at the very least we want some bulk calculation to go out of control when we ask for the density of states $\rho(E)$.
- In many ways this is a small detail, but it may help with other problems: e.g.
  - BH information problem ('t Hooft, Maldacena, Strominger and others have emphasized this aspect of BHIP).
  - Is de Sitter a normal quantum system?

Possible analogy [Berry, Keating]

Let $\{s_n\}$ be the nontrivial zeros of $\zeta(\frac{1}{2}+is)$, and consider

$$Z(\beta) = \sum_n e^{-\beta E_n}, \qquad E_n = s_n^2.$$

Manipulating the product-over-primes formula gives [Guinand]

$$Z(\beta) = e^{\frac{\beta}{4}} + \int_0^\infty ds \left[\rho_{smooth}(s) - \frac{1}{\pi}\sum_{p^m} \frac{\log(p)}{p^{m/2}} \cos(s\, m \log\, p)\right] e^{-\beta s^2}$$
$$= e^{\frac{\beta}{4}} + Z_{smooth}(\beta) + e^{-\frac{\log(2)^2}{4\beta}} + e^{-\frac{\log(3)^2}{4\beta}} \ldots.$$

For $Z(\beta)$, corrections are extremely small once β is small enough to include even just the first energy level with $O(1)$ coefficient.

However, for $\rho(s)$ (the thing in brackets), the sum diverges.

Is there an analog of $\sum_{p^m}$ in the bulk theory? [Shenker,...]