Swampland, field distances and naturalness



Irene Valenzuela



Utrecht University

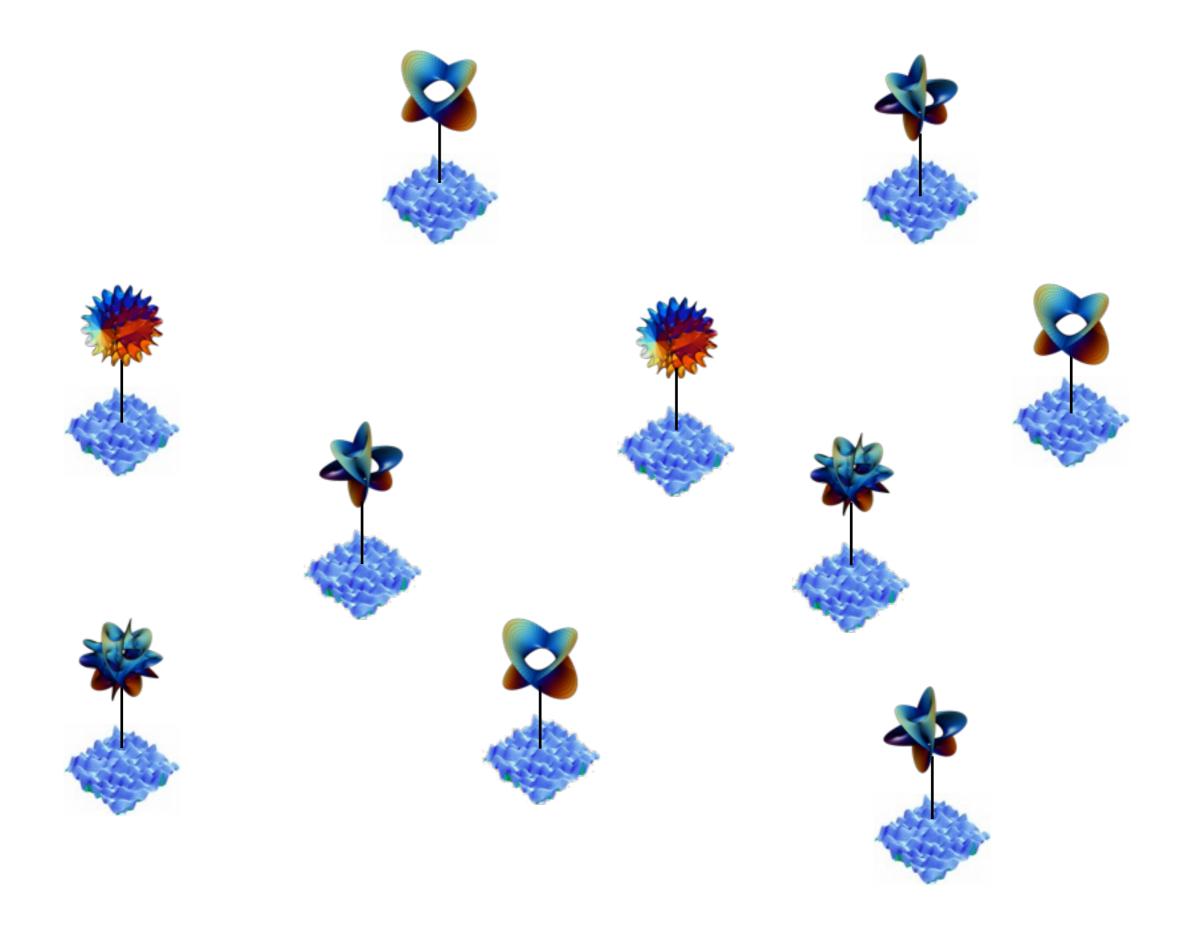
Grimm, Palti, IV [arXiv:1802.08264 [hep-th]]

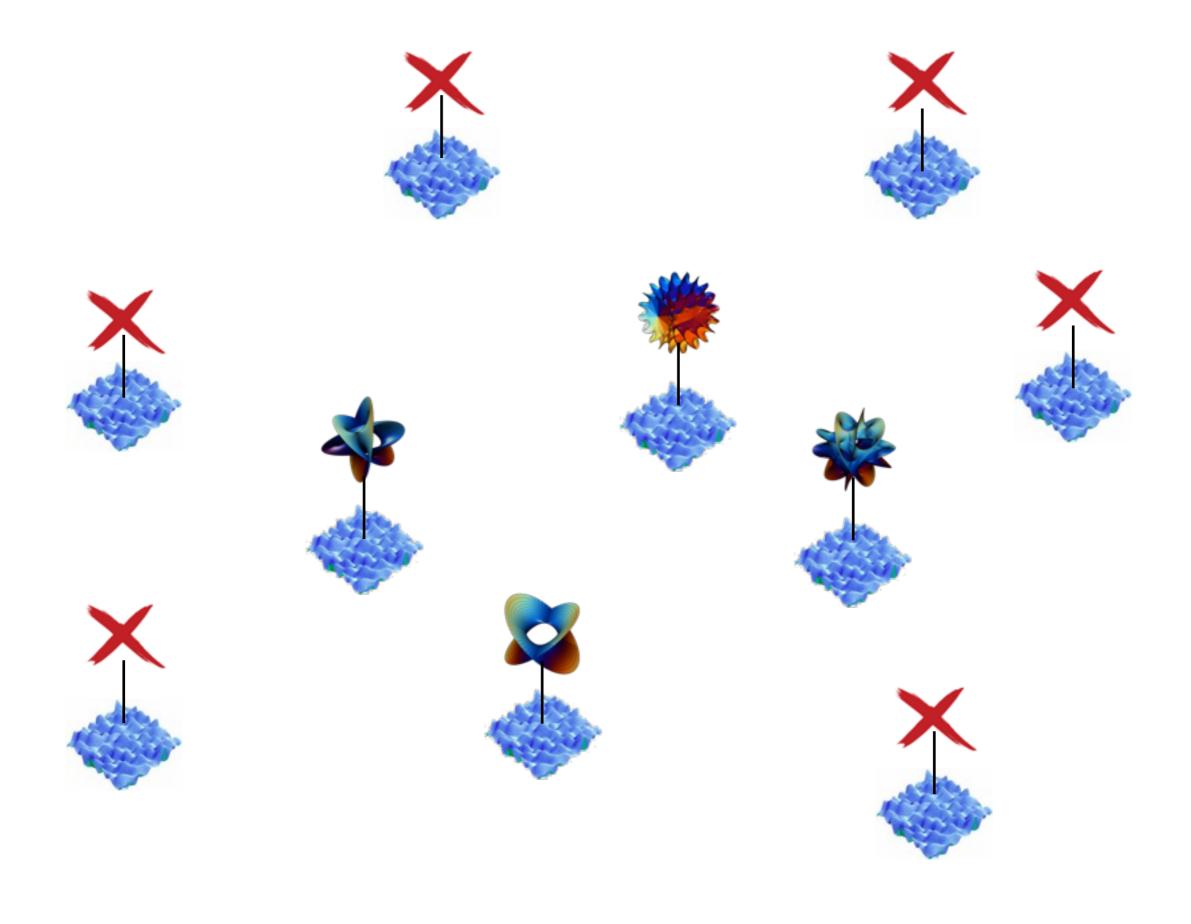
Ibanez, Martin-Lozano, IV [arXiv: 1707.05811 [hep-th]] Ibanez, Martin-Lozano, IV [arXiv: 1706.05392 [hep-th]]

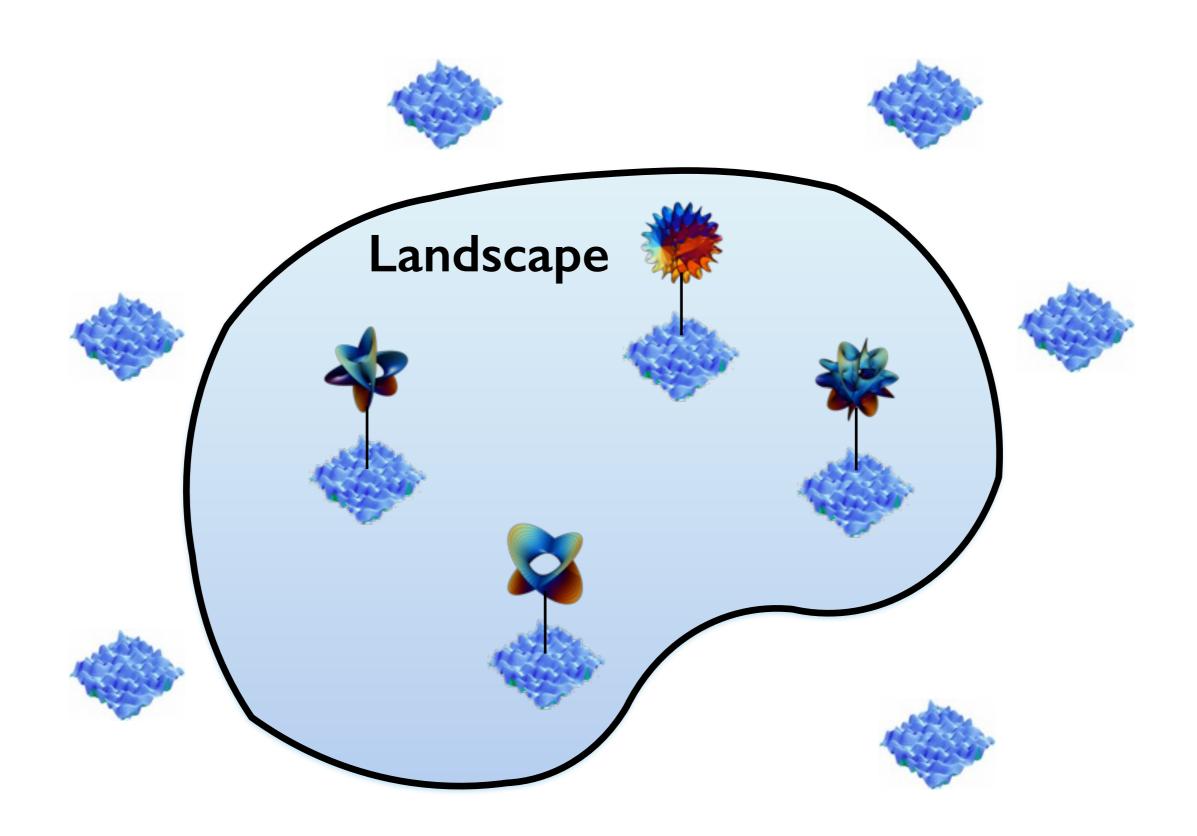
Strings 2018, Okinawa (Japan)

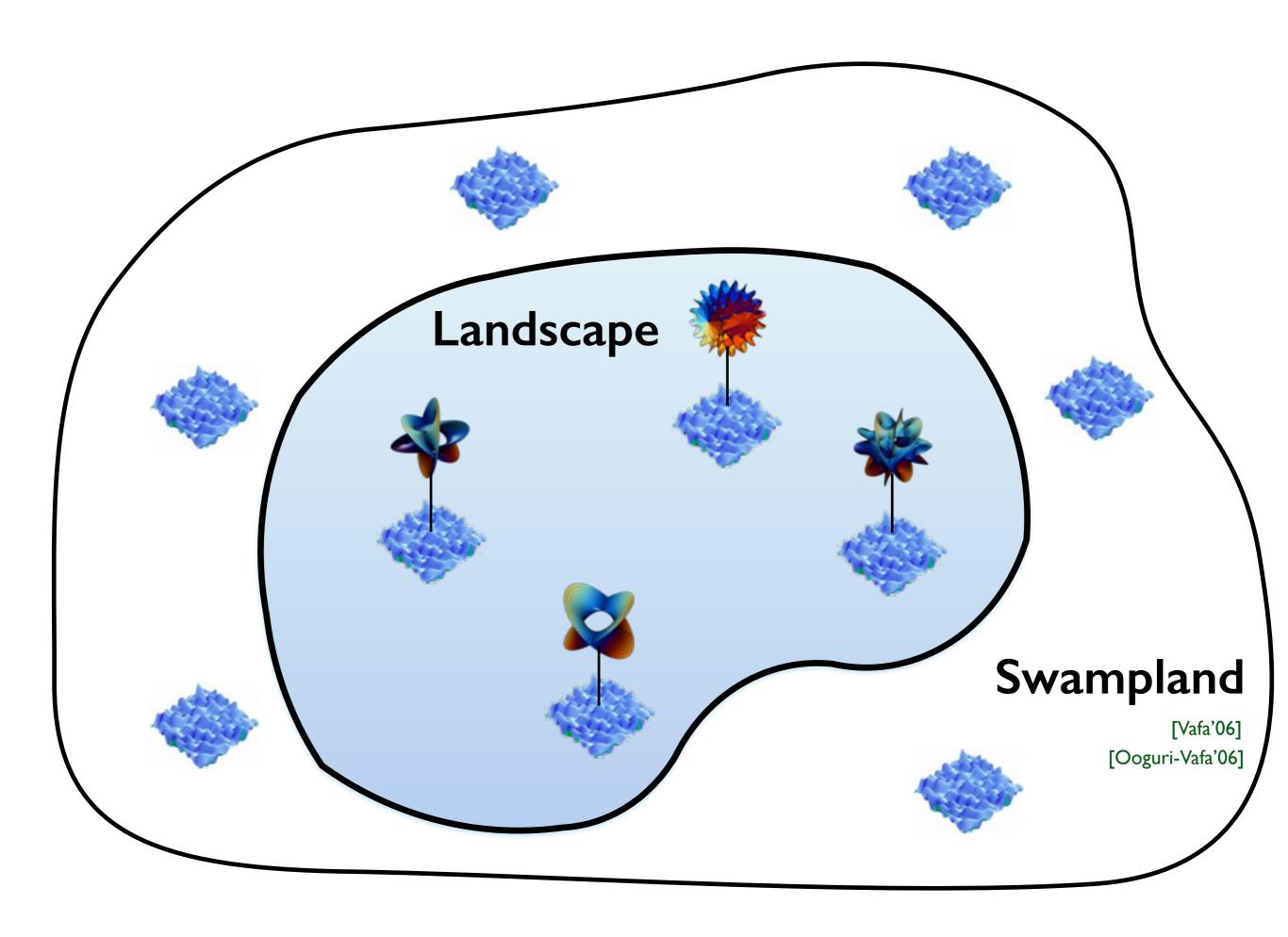
Swampland:

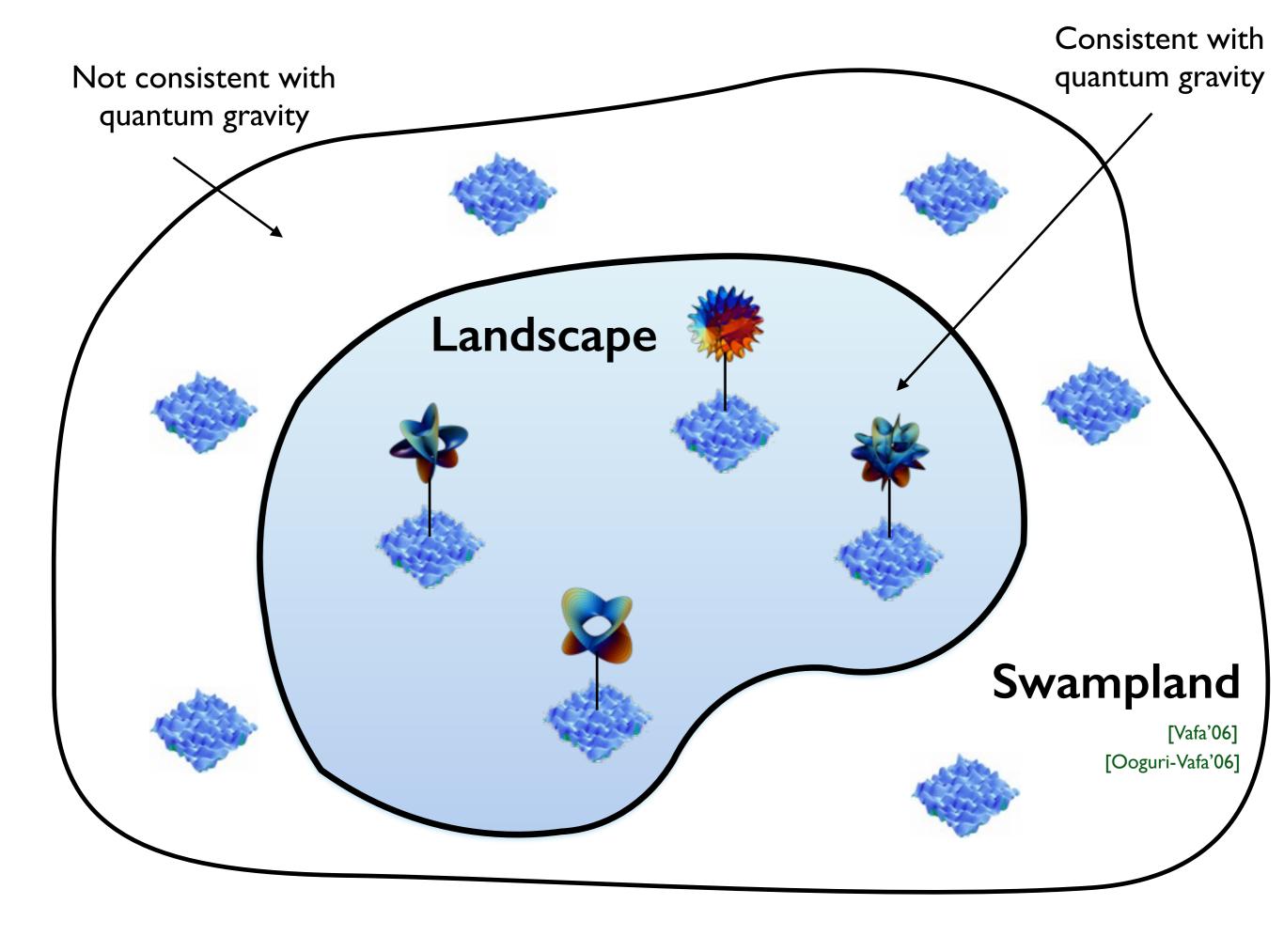
Apparently consistent (anomaly-free) quantum effective field theories that cannot be UV embedded in quantum gravity (they cannot arise from string theory)











Not everything is possible in string theory/quantum gravity!!!

Additional QG constraints = UV imprint at low energies

= Quantum Gravity/String Theory predictions!

String Theory

Phenomenology

What are the constraints that an effective theory must satisfy to be consistent with quantum gravity?

What distinguishes the landscape from the swampland?

Motivated by observing recurrent features of the string landscape as well as black hole physics

- Absence of global symmetries [Banks-Dixon'88] [Horowitz,Strominger,Seiberg...]
- Completeness hypothesis [Polchinski.'03]
- Weak Gravity Conjecture [Arkani-Hamed et al.'06]
- Swampland Distance Conjecture [Ooguri-Vafa'06]
- No stable non-susy AdS vacua [Ooguri-Vafa'16]
- No deSitter vacuum?

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Large field inflation

→ Particle physics and c.c.

They can have significant implications in low energy physics!

- UV sensitive effective theories
- Naturalness issues

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Compactifications of the Standard Model

Ooguri-Vafa conjecture: non-susy stable AdS vacua are not consistent with QG

Problem: AdS vacua can arise when compactifying SM to lower dimensions (they might be stable if 4d bubbles are too big to be transferred to 3d)

Compactifications of the Standard Model

Ooguri-Vafa conjecture: non-susy stable AdS vacua are not consistent with QG

Problem: AdS vacua can arise when compactifying SM to lower dimensions (they might be stable if 4d bubbles are too big to be transferred to 3d)

Solution:

Avoid presence of AdS vacua



Constraints on light spectra of SM!

(Explain SM hierarchies!)

Upper bound on neutrino masses in terms of the cosmological constant

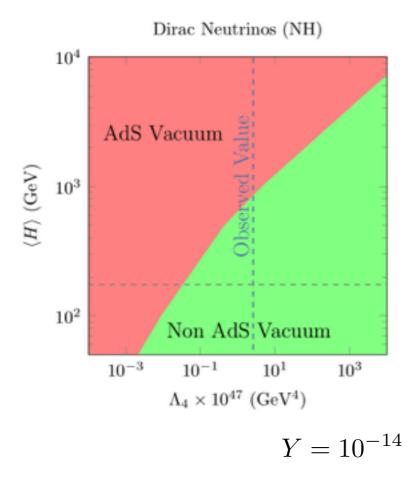
$$\sum_i m_{\nu_i}^4 \lesssim \mathcal{O}(\Lambda_4)$$
 (Upper bound on EW scale!) [Ibanez,Martin-Lozano,IV'17] (see also [Hamada-Shiu'17])

- Constraints on BSM spectra [Ibanez, Martin-Lozano, IV' 17]
- SM by itself ruled out --- MSSM survives [Gonzalo, Herraez, Ibanez' 18]

Naturalness?

Consistency with quantum gravity requires $\langle H \rangle \lesssim 1.6 \frac{\Lambda^{1/4}}{Y_{\nu_1}}$ [Ibanez, Martin-Lozano, IV' 17]

(recall: sufficient but not necessary condition)



Parameters leading to a higher EW scale do not yield theories consistent with quantum gravity



New approach to fine-tuning/hierarchy problems? UV/IR mixing from quantum gravity?

Naturalness might not be a good principle, not everything goes!

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Very important to gather more evidence to prove (or disprove) them

—— doable task

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doable task

- Definition and implications
- Test in the complex structure moduli space of Type IIB CY compactifications
- General insights

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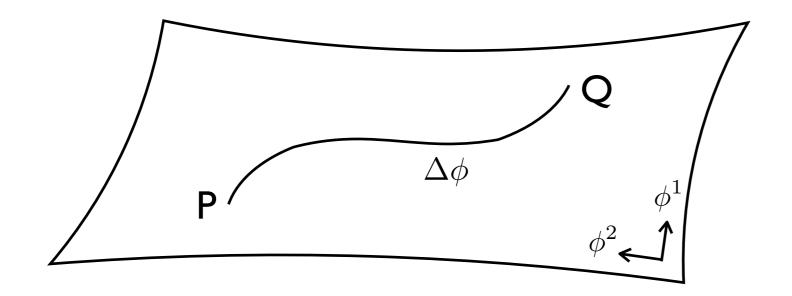
An effective theory is valid only for a finite scalar field variation $\Delta\phi$

because an infinite tower of states become exponentially light

$$m \sim m_0 e^{-\lambda \Delta \phi}$$
 when $\Delta \phi \to \infty$

Consider the moduli space of an effective theory:

$$\mathcal{L} = g_{ij}(\phi)\partial\phi^i\partial\phi^j$$
 scalar manifold



$$\Delta \phi = {
m geodesic\ distance}$$
 between P and Q

$$m(P) \lesssim m(Q)e^{-\lambda\Delta\phi}$$

Swampland Distance Conjecture [Ooguri-Vafa'06]

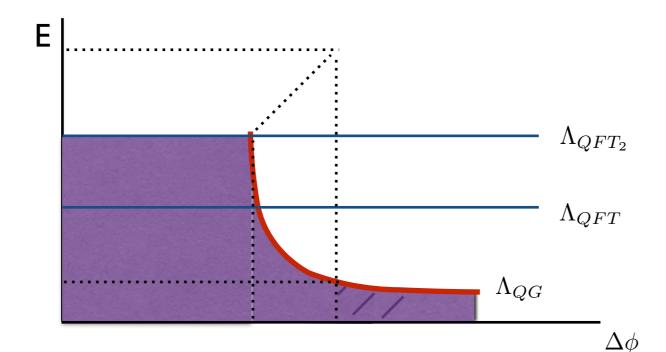
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because an infinite tower of states become exponentially light

$$m \sim m_0 e^{-\lambda \Delta \phi}$$
 when $\Delta \phi \to \infty$

This signals the breakdown of the effective theory:

$$\Lambda_{\text{cut-off}} \sim \Lambda_0 \exp(-\lambda \Delta \phi)$$



It gives an upper bound on the scalar field range described by any effective field theory with finite cut-off

$$\Delta \phi \lesssim \frac{1}{\lambda} \log \left(\frac{M_p}{E} \right)$$

Phenomenological implications:

- Large field inflation
- Cosmological relaxation of the EW scale
- It applies to any scalar (also axions realising axion monodromy in IIB [Baume,Palti'16] string theory upon taking into account back-reaction on kinetic term) [I.V.,'16]
- Examples compatible with the Refined SDC: [Ooguri-Vafa'06] [Klaewer,Palti'16]

$$\lambda \sim 1 \rightarrow \Delta \phi \lesssim \mathcal{O}(1) M_p$$

Caveats!

non-geodesics?

mass hierarchies?

Evidence: based on particular examples in string theory compactifications

[Ooguri, Vafa'06] [Baume, Palti'16] [I.V.,'16] [Bielleman, Ibanez, Pedro, I.V., Wieck'16] [Blumenhagen, I.V., Wolf'17] [Hebecker, Henkenjohann, Witkowski'17] [Cicoli, Ciupke, Mayhrofer, Shukla'18] [Blumenhagen et al.'18]

- Model-independent understanding missing...
- Very little is known about the tower of particles...
- What is the underlying QG obstruction?

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[Grimm, Palti, IV.' 18]

Focus: Complex structure moduli space of IIB CY₃ compactifications (4d N=2 string theory moduli space preserving special Kahler geometry)



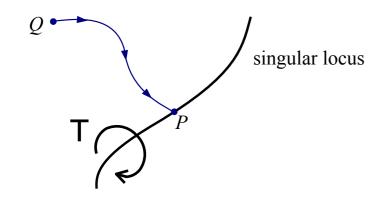
natural testing ground for QG

Aim: Prove the conjecture for any infinite distance path!

- Definition and implications
- Test in the complex structure moduli space of Type IIB CY compactifications
- General insights

Complex structure moduli space of IIB CY compactifications

Prime example of a field space capturing information about 'quantum gravity'



Infinite geodesic distances can occur only if approaching a singularity

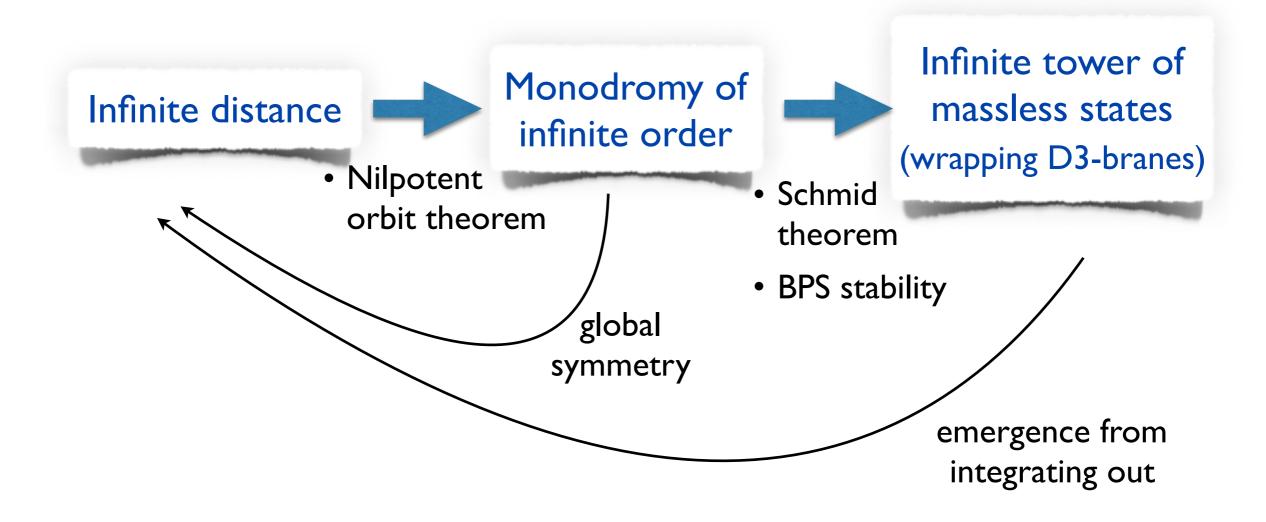
Massless BPS states (wrapping D3-branes) arise at the singularities

Candidates for SDC tower!

Two types:

- Infinite distance singularities: any trajectory approaching P has infinite length
- Finite distance singularities: at least one trajectory approaching P has finite length

Aim: Identify infinite tower of exponentially massless BPS states at any infinite distance singularity



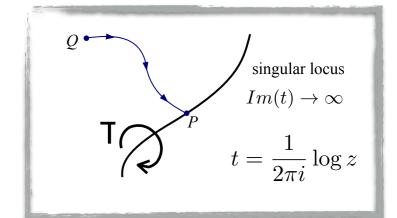
[Grimm,Palti,IV.'18]

Focus on points belonging to a single singular divisor $\sqrt{}$

• Distances given by:
$$d_{\gamma}(P,Q) = \int_{\gamma} \sqrt{g_{IJ}\dot{x}^I\dot{x}^J}ds$$

$$g_{I\bar{J}} = \partial_{z^I} \partial_{\bar{z}^J} K$$

$$K = -\log \left(-i^D \int_{Y_D} \Omega \wedge \bar{\Omega} \right)$$



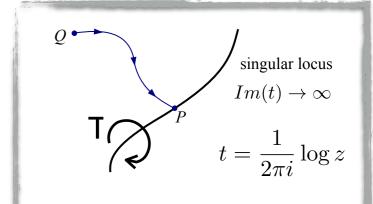
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• Periods of the (D,0)-form:
$$\Pi^{\mathcal{I}} = \int_{\Gamma_{\mathcal{I}}} \Omega$$

transform under monodromy $\Pi(e^{2\pi iz}) = T \cdot \Pi(z)$ (remnant of higher dimensional gauge symmetries)



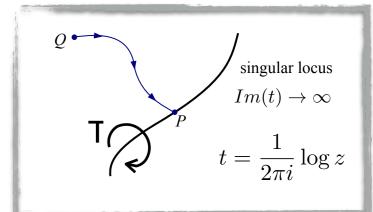
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• Define nilpotent matrix $N = \log T$ (only non-zero if monodromy T is of infinite order) (no k s.t. $T^k = T$)

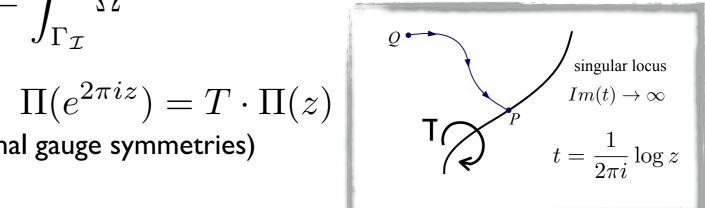
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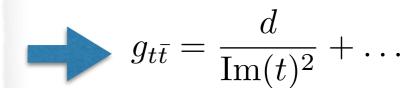


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Nilpotent orbit theorem:

[Schmid'73]

$$\Pi(t,\eta) = \exp(tN) a_0(\eta) + \mathcal{O}(e^{2\pi it},\eta)$$



It gives local expression for the periods near singular locus!

$$\Pi(t,\eta) = (1 + tN + \dots + t^d N^d) a_0(\eta) + \mathcal{O}(e^{2\pi i t,\eta})$$

1) Infinite distances only if monodromy is of infinite order

Theorem:

P is at infinite distance



[Wang'97, Lee'16]

$$d_{\gamma}(P,Q) = \sqrt{d} \log(\operatorname{Im} t)|_{Q}^{P} \to \infty$$

1) Infinite distances only if monodromy is of infinite order

Theorem: P is at infinite distance $Na_0 \neq 0$ [Wang'97, Lee'16] $d_{\gamma}(P,Q) = \sqrt{d} \log(\operatorname{Im} t)|_{Q}^{P} \to \infty$

2) Monodromy can be used to populate an infinite orbit of BPS states

Mass given by central charge: $Z=e^Kq\cdot\Pi$ $q=(q_e^I,q_I^m)$ $\lim_{\substack{q_1 \ q_2 \ q_3 \ q_4 \ q_5}} \overline{q_m} = T^m q \qquad m \in \mathbb{Z}$



If T is of infinite order Starting with one state, we generate infinitely many!

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If T is of infinite order Starting with one state, we generate infinitely many!

3) Universal local form of the metric gives the exponential mass behaviour

$$Z = \frac{1}{t^{d/2}} \sum_{j=1}^{d} (q \cdot N^j a_0) t^j + \mathcal{O}(e^{2\pi i t}) \qquad \qquad \frac{M_q(P)}{M_q(Q)} \simeq \exp\left(-\frac{1}{\sqrt{2d}} d_\gamma(P, Q)\right)$$

Massless: $q^T N^j a_0 = 0$, j > d/2

Infinite massless monodromy orbit at the singularity



Infinite tower of states becoming exponentially light





Swampland Distance Conjecture (SDC) is reduced to prove the existence of an infinite massless monodromy orbit at the singularity

$$\exists \, q \; ext{ s.t. } \; q^T N^j a_0 = 0 \, , \quad j \geq d/2 \quad ext{(Massless)}$$
 $Nq \neq 0 \quad ext{(Infinite orbit)}$

Tool: mathematical machinery of mixed hodge structure (introduce finer split of cohomology at the singularity adapted to N)

[Deligne][Schmid][Cattani, Kaplan, Schmid] [Kerr, Pearlstein, Robles' 17]

(Subtleties regarding stability of BPS states: need to mod out states with $q^T N^j a_0 = 0$, $\forall j$)

BPS states and stability

How many BPS states are when approaching singularity?

Do they cross a wall of marginal stability upon circling the monodromy locus?

Consider:
$$\mathbf{q}_C = \mathbf{q}_B + \mathbf{q}_{\bar{A}}$$
 \longrightarrow $M_{\mathbf{q}_C} \leq M_{\mathbf{q}_B} + M_{\mathbf{q}_{\bar{A}}}$



$$M_{\mathbf{q}_C} \le M_{\mathbf{q}_B} + M_{\mathbf{q}_{\bar{A}}}$$

Wall of marginal stability:
$$\varphi\left(B\right)-\varphi\left(A\right)=1$$
 with $\varphi\left(A\right)=\frac{1}{\pi}\mathrm{Im}\log Z_{\mathbf{q}_{A}}$

Upon circling the monodromy locus n times:

$$\varphi_{\rm I} \to \varphi_{\rm I} - \frac{n}{\pi {
m Im}\,t} + \mathcal{O}\left(\frac{1}{\left({
m Im}\,t\right)^2}\right)$$

unstable
$$\begin{array}{c|c} & \vdots & \\ \hline T^n q_0 & \underline{ } \\ & \\ \text{stable} \end{array} \begin{array}{c|c} & \vdots & \\ \hline Tq_0 & \underline{ } \\ & \\ \hline & \\ \end{array}$$

Charge states $\mathbf{q} = T^n \mathbf{q}_0$ with $n \ll \operatorname{Im} t$ are stable (grade does not change) (states higher up in the tower are unstable)

Number of BPS states: $n \sim \text{Im}(t) \sim e^{d_{\gamma}(P,Q)}$

(grows when approaching the singularity and diverges there)

Swampland Distance Conjecture

- Definition and implications
- Test in the complex structure moduli space of Type IIB CY compactifications
- General insights

Global symmetries

SDC as a quantum gravity obstruction to restore a global continuous axionic shift symmetry at the singular point

$$K = -\log[p_d(\operatorname{Im} t) + \mathcal{O}(e^{2\pi i t})]$$

At infinite distance singularities: $\operatorname{Re} t \to \operatorname{Re} t + c$, $c \in \mathbb{R}$ when $\operatorname{Im} t \to \infty$

 $\operatorname{Re} t$ = axion with decay constant $f^2 = g_{t\bar{t}} \to 0$

(also, gauge coupling of dual 2-form gauge field goes to zero)

→ analogous to WGC

Global symmetries

SDC = Magnetic Scalar WGC

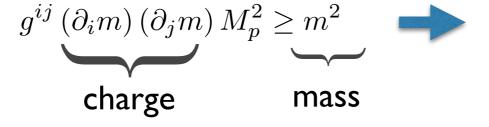
• Magnetic version:

WGC: $\Lambda < gM_p$ If $g \to 0$ global symmetry is restored How small can the gauge coupling be?

SDC: $\Lambda \sim M_p \, \exp(-\lambda \Delta \phi)$ If $\Delta \phi \to \infty$ global symmetry is restored

How large can the field variation be?

Electric version:





satisfied for long distance if mass is exponential in ϕ [Palti'17]

These moduli spaces are 'quantum in nature'



geometry incorporates information about integrating out BPS states

Conifold singularity: log-divergence of gauge coupling from integrating [Strominger'95] out a single BPS D3-state

Similar computation at infinite distance singularities! [Grimm,Palti,IV.'18]

One-loop corrections from integrating out the tower of BPS states



matches geometric result (generates the log. infinite distance)

At infinite distance singularities:

$$g_{t\bar{t}} = \frac{d}{\operatorname{Im}(t)^2} + \dots$$

$$d_{\gamma}(P,Q) = \int_{Q}^{P} \sqrt{g_{t\bar{t}}} |dt| \sim \frac{\sqrt{2}}{2} \log(\operatorname{Im} t)|_{Q}^{P}$$

$$\Delta m \left\{ egin{array}{c} \Lambda_{UV} = \Lambda_{
m Species} \ m_2 \ m_1 \ m_0 = \Lambda_0 \ \end{array}
ight.$$

Original theory:

$$\mathcal{L} = \frac{1}{2} (\partial \phi)^2 + \sum_{i=1}^{S} \left[\frac{1}{2} (\partial h_i)^2 + \frac{1}{2} m_i (\phi)^2 h_i^2 \right]$$
light field tower of massive fields them out!
$$m_k = m_0 + k \Delta m$$
integrate them out!

We have to integrate out the tower of particles up to the UV cut-off of the original theory!

$$\Lambda_{UV} = \frac{M_p}{\sqrt{S}}$$
 [Dvali'07]

ightharpoonup number of fields below Λ_{UV}

$$S = \frac{\Lambda_{UV}}{\Delta m(\phi)}$$

$$\Delta m(\phi) \sim \phi^{-d/2}$$

$$\Lambda_{UV}(\phi) \sim \Delta m(\phi)^{1/3} \sim \phi^{-d/6}$$

Field dependent UV cut-off!

At the singularity: $S \to \infty \Rightarrow \Lambda_{UV} \to 0$

(growth of S matches with stability of BPS states!)

$$\Delta m \left\{ egin{array}{c} \Lambda_{UV} = \Lambda_{
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integrate them out!

Quantum correction to the field metric — matches geometric result

$$\delta g_{\phi\phi} \propto \sum_{k=1}^{S} (\partial_{\phi} m_k)^2 \sim \frac{d}{\phi^2}$$
 $d(\phi_1, \phi_2) \sim \sqrt{d} \log \left(\frac{\phi_2}{\phi_1}\right)$

Quantum correction to the gauge kinetic function —— matches geometric result

$$g_{YM}^2 \sim \phi^{-n} \sim m_0^{2n}$$

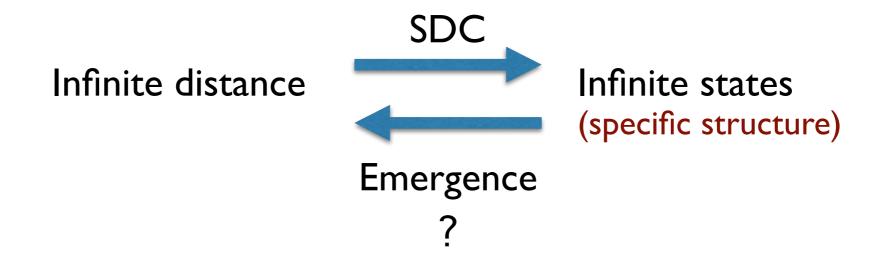
UV cut-off decreases exponentially fast in the proper field distance $\Lambda \sim M_p \, e^{-\lambda \, d(\phi_1, \phi_2)}$

effective theory completely breaks down SDC! ✓



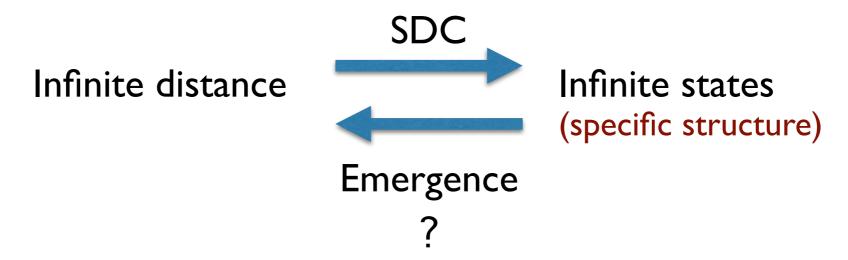
Infinite distance and weak coupling emerge from integrating out an infinite tower of states!

Can this be a general feature for any moduli space?



Infinite distance and weak coupling emerge from integrating out an infinite tower of states!

Can this be a general feature for any moduli space?



Comments:

 $m(\phi)$ does not matter much as long as $S(\phi) \neq const.$ is given by species bound of a tower of particles and $\frac{\partial_{\phi} m}{m} \gtrsim \mathcal{O}\left(\frac{1}{\phi}\right)$ when $\phi \to \infty$

(see also [Heidenreich, Reece, Rudelius' 18])

Infinite distance and weak coupling emerge from integrating out an infinite tower of states!

Can this be a general feature for any moduli space?

This limits correspond to restoring a continuous global symmetry, so global symmetries would also be emergent from integrating out infinitely many states! (emergence is continuous)

$$\Lambda_{UV} = \frac{M_p}{\sqrt{S}} \longrightarrow \mathbf{0}$$
 when $S \to \infty$ unless $M_p \to \infty$

Global symmetries only possible if gravity decouples

Summary

Swampland Distance Conjecture:

Upper bound on the scalar field range: Implications for inflation!

- √ Test in the complex structure moduli space of CY IIB compactifications
 - Infinite order monodromy as generator of the infinite tower
 - Emergence of infinite field distance and global symmetry

Generalizations:

- Our results are valid for any CY (model-independent)
 (but only for infinite distance points that belong to a single singular divisor)
- Other moduli spaces?

Summary

Consistency with quantum gravity implies constraints on low energy physics.

Knowledge of Swampland is essential for UV sensitive theories and might also be important for naturalness issues.

Very important to gather more evidence to prove or disprove the conjectures

doable and exciting task!

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Consistency with quantum gravity implies constraints on low energy physics.

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Thank you!

back-up slides

Infinite tower of states

Tool: mathematical machinery of mixed hodge structure

Problem: 'Normal' Hodge decomposition no longer useful when approaching a singularity $H^3(Y_3,\mathbb{C})=H^{3,0}\oplus H^{2,1}\oplus H^{1,2}\oplus H^{0,3}$

Idea: Introduce finer split of cohomology at the singularity adapted to this 'limiting' Hodge decomposition $h^{p,3-p} = \sum \dim I^{p,q}$

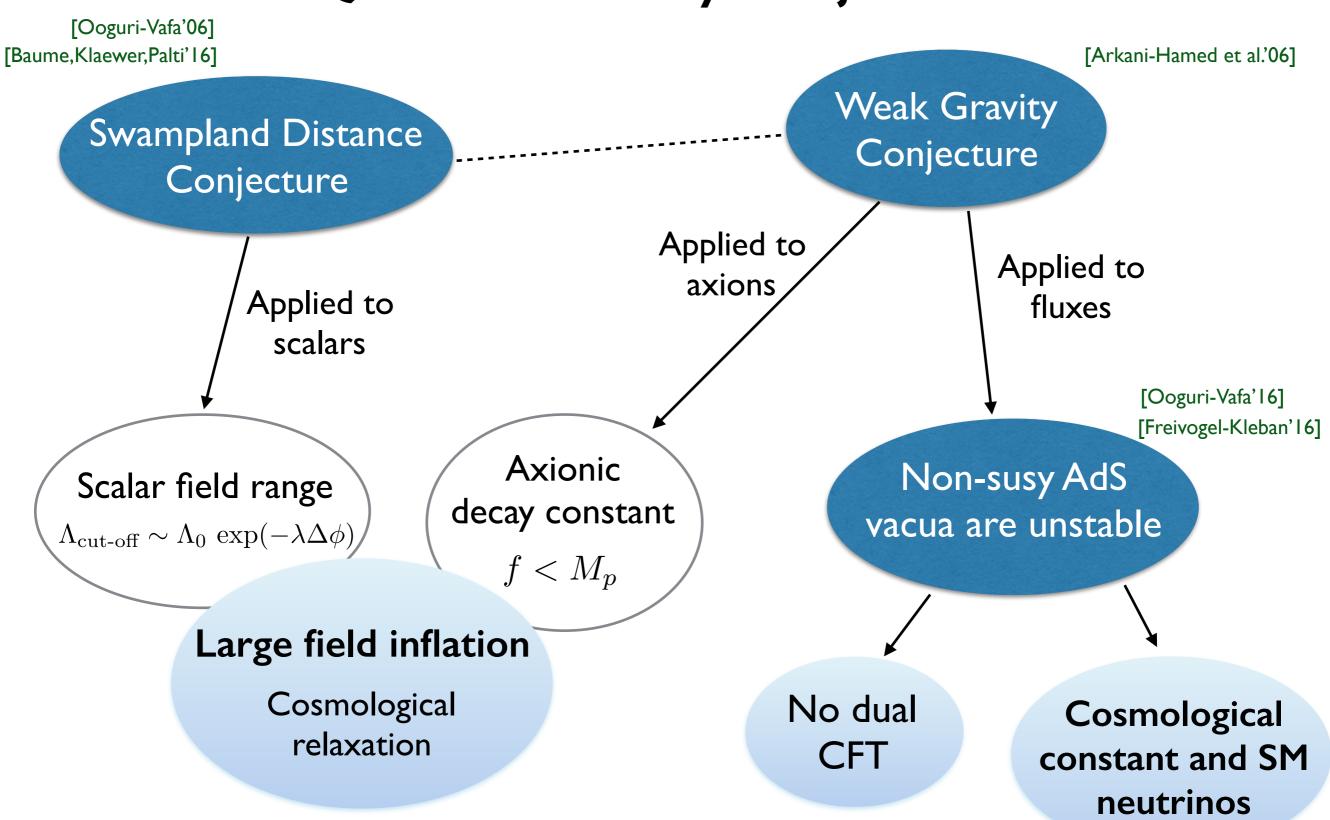
$$H^{3,0} \rightarrow \{I^{3,3}, I^{3,2}, I^{3,1}, I^{3,0}\}$$

[Deligne][Schmid][Cattani,Kaplan,Schmid] [Kerr,Pearlstein,Robles'17]

The subspaces capture non-trivial information about the nilpotent monodromy operator, $NI^{p,q}\subset I^{p-1,q-1}$

For a CY: $\mathbf{a}_0 \in I^{3,d}$, d = 0, 1, 2, 3

Quantum Gravity Conjectures



Casimir energy

Potential energy in 3d:

$$V(R) = \frac{2\pi r^3 \Lambda_4}{R^2} + \sum_{i} (2\pi R) \frac{r^3}{R^3} (-1)^{s_i} n_i \rho_i(R)$$

Casimir energy density:

$$\rho(R) = \mp \sum_{n=1}^{\infty} \frac{2m^4}{(2\pi)^2} \frac{K_2(2\pi Rmn)}{(2\pi Rmn)^2}$$

For small mR:

$$\rho(R) = \mp \left[\frac{\pi^2}{90(2\pi R)^4} - \frac{\pi^2}{6(2\pi R)^4} (mR)^2 + \frac{\pi^2}{48(2\pi R)^4} (mR)^4 + \mathcal{O}(mR)^6 \right]$$

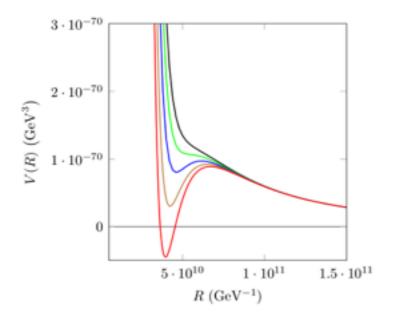
Compactifications of the Standard Model

Standard Model + Gravity on S^1 :)

[Arkani-Hamed et al.'07] (also [Arnold-Fornal-Wise'10])

$$V(R) = \frac{2\pi\Lambda_4}{R^2}$$
 + Casimir energy tree-level one-loop correct





We can get AdS, Minkowski or dS vacua in lower dimensions!

(depending on light spectra of SM and value of c.c.)

Compactifications of the Standard Model

Problem:

AdS minimum appears if:

- Neutrinos are Majorana
- Neutrinos have a Dirac mass: $\sum_i m_{
 u_i}^4 \gtrsim \mathcal{O}(\Lambda_4)$

For our c.c: $m_{\nu_1} > 7.7 \,\, \mathrm{meV} \,\, \mathrm{(NH)}$ $m_{\nu_1} > 2.1 \,\, \mathrm{meV} \,\, \mathrm{(IH)}$

[lbanez,Martin-Lozano,IV'17]

Assuming Ooguri-Vafa AdS conjecture + background independence:

We should not get stable non-susy AdS vacua from compactifying the SM !!!

Compactifications of the Standard Model

Solution:

Make the minimum unstable

- Include Wilson Lines to generate a runaway for small R [Hamada-Shiu'17]
 - → not valid in orbifolds
- Assume existence of 4d bubble instability which is transferred to lower dimensions
 - \longrightarrow not valid if $R_{\text{buble}} > l_{AdS_3}$

[lbanez,Martin-Lozano,IV'17]

Impose absence of AdS vacua

Constraints on light spectra of SM!

Upper bound on neutrino masses in terms of the cosmological constant

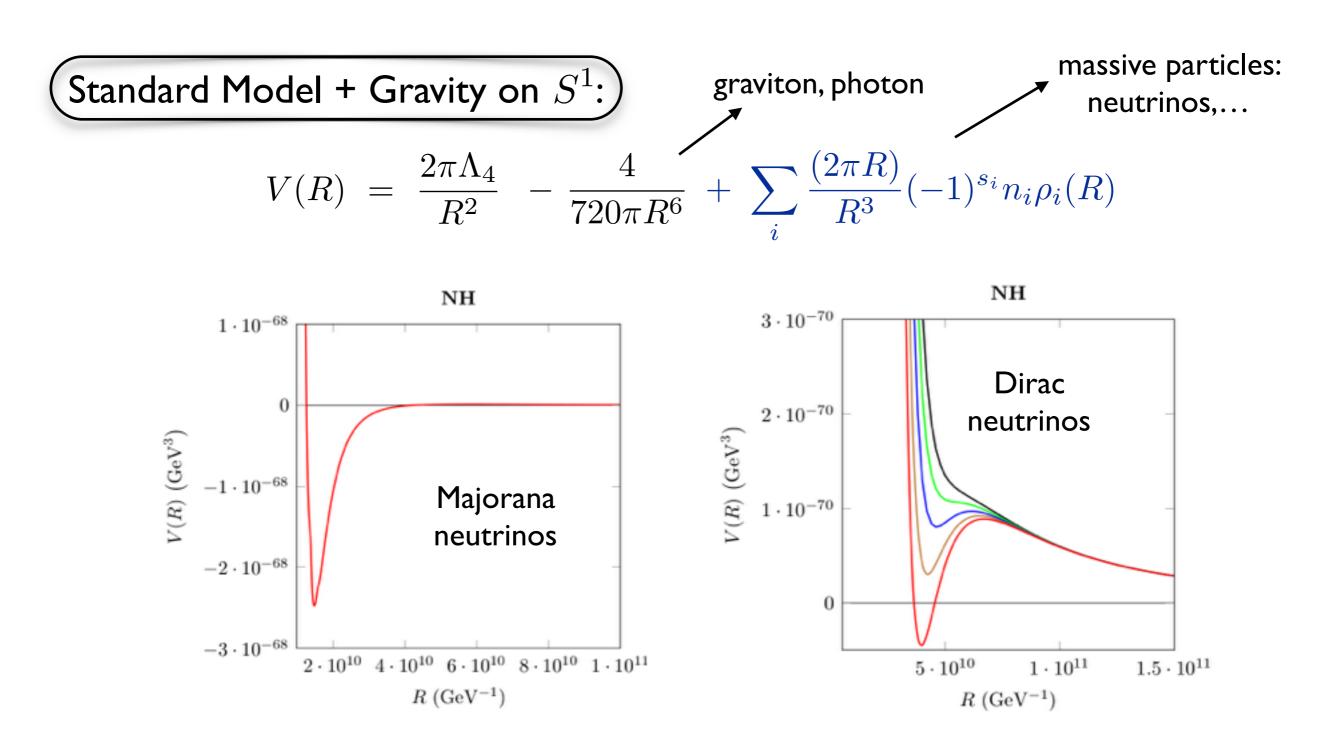
$$\Lambda_4 \gtrsim \mathcal{O}(m_{\nu}^4)$$

(Upper bound on EW scale!)

§ SM by itself ruled out → MSSM survives

[Gonzalo,Herraez,Ibanez'18]

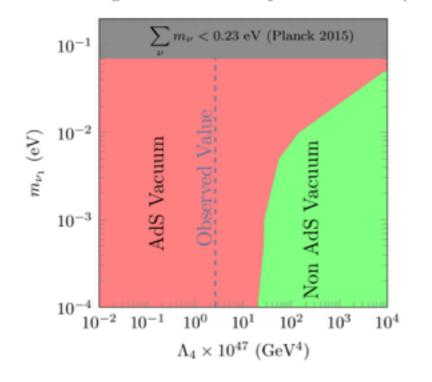
Compactification of the SM to 3d



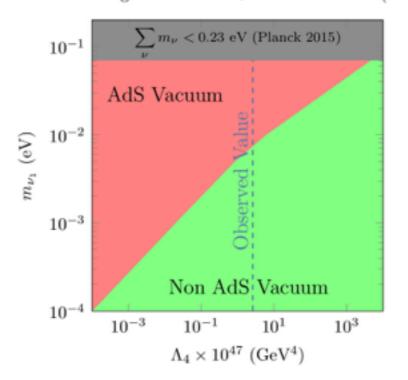
The more massive the neutrinos, the deeper the AdS vacuum

Lower bound on the cosmological constant

Cosmological Constant + Majorana Neutrinos (NH)



Cosmological Constant + Dirac Neutrinos (NH)



The bound for Λ_4 scales as $m_{
u}^4$

(as observed experimentally)

$$\Lambda_4 \ge \frac{a(n_f)30(\Sigma m_i^2)^2 - b(n_f, m_i)\Sigma m_i^4}{384\pi^2}$$

with
$$a(n_f) = 0.184(0.009) \ b(n_f, m_i) = 5.72(0.29)$$
 for Majorana (Dirac)

First argument (not based on cosmology) to have $\Lambda_4 \neq 0$

Adding BSM physics

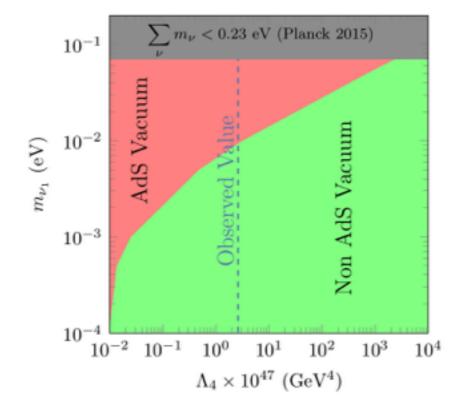
Light fermions

Positive Casimir contribution — helps to avoid AdS vacuum

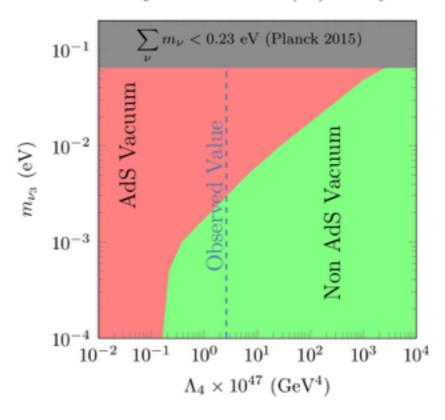
Majorana neutrinos are consistent if adding $m_\chi \lesssim 2 \,\,\mathrm{meV}$

example. For $m_\chi = 0.1~{\rm meV}$:

C.C. + Majorana Neutrinos (NH) + Weyl fermion



C.C. + Majorana Neutrinos (IH) + Weyl fermion

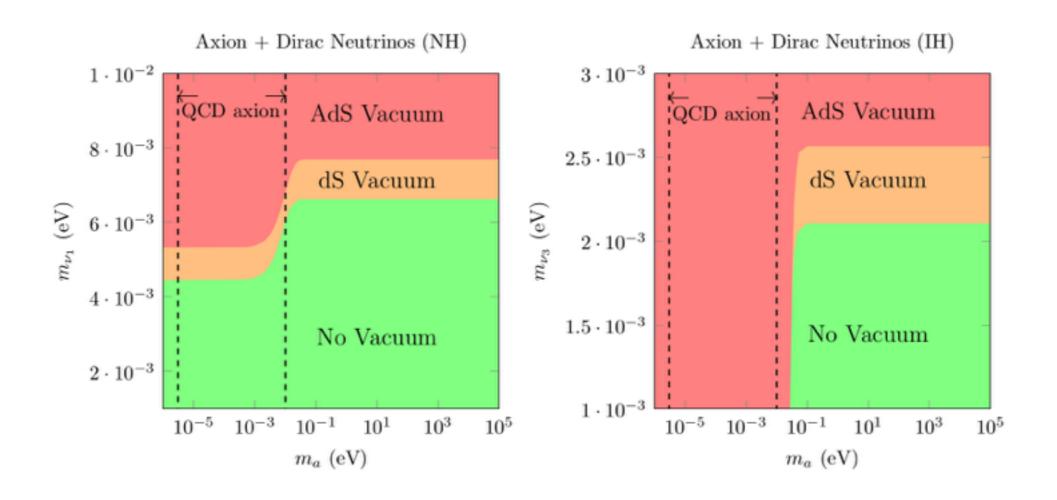


Adding BSM physics

Axions

1 axion: negative contribution — bounds get stronger

Multiple axions: can destabilise AdS vacuum



Bounds on the SM + light BSM physics

Model	Majorana (NI)	Majorana (IH)	Dirac (NH)	Dirac (IH)
SM(3D)	no	no	$m_{\nu_1} \le 7.7 \times 10^{-3}$	$m_{\nu_3} \le 2.56 \times 10^{-3}$
SM(2D)	no	no	$m_{\nu_1} \le 4.12 \times 10^{-3}$	$m_{\nu_3} \le 1.0 \times 10^{-3}$
SM+Weyl(3D)	$m_{\nu_1} \le 0.9 \times 10^{-2}$	$m_{\nu_3} \le 3 \times 10^{-3}$	$m_{\nu_1} \le 1.5 \times 10^{-2}$	$m_{\nu_3} \le 1.2 \times 10^{-2}$
	$m_f \le 1.2 \times 10^{-2}$	$m_f \le 4 \times 10^{-3}$		
SM+Weyl(2D)	$m_{\nu_1} \le 0.5 \times 10^{-2}$	$m_{\nu_3} \le 1 \times 10^{-3}$	$m_{\nu_1} \le 0.9 \times 10^{-2}$	$m_{\nu_3} \le 0.7 \times 10^{-2}$
	$m_f \le 0.4 \times 10^{-2}$	$m_f \le 2 \times 10^{-3}$		
SM+Dirac(3D)	$m_f \le 2 \times 10^{-2}$	$m_f \le 1 \times 10^{-2}$	yes	yes
SM+Dirac(2D)	$m_f \le 0.9 \times 10^{-2}$	$m_f \le 0.9 \times 10^{-2}$	yes	yes
$SM+1 \operatorname{axion}(3D)$	no	no	$m_{\nu_1} \le 7.7 \times 10^{-3}$	$m_{\nu_3} \le 2.5 \times 10^{-3}$
				$m_a \ge 5 \times 10^{-2}$
SM+1 axion(2D)	no	no	$m_{\nu_1} \le 4.0 \times 10^{-3}$	$m_{\nu_3} \le 1 \times 10^{-3}$
				$m_a \ge 2 \times 10^{-2}$
$\geq 2(10)$ axions	yes	yes	yes	yes

Compactifications of SM on T_2 — qualitatively similar, but a bit stronger

(see also [Hamada-Shiu'17])