REVIEW:

Bounds on Energy, Entropy, and Transport

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Various local energy conditions are often discussed in classical general relativity:

Strong

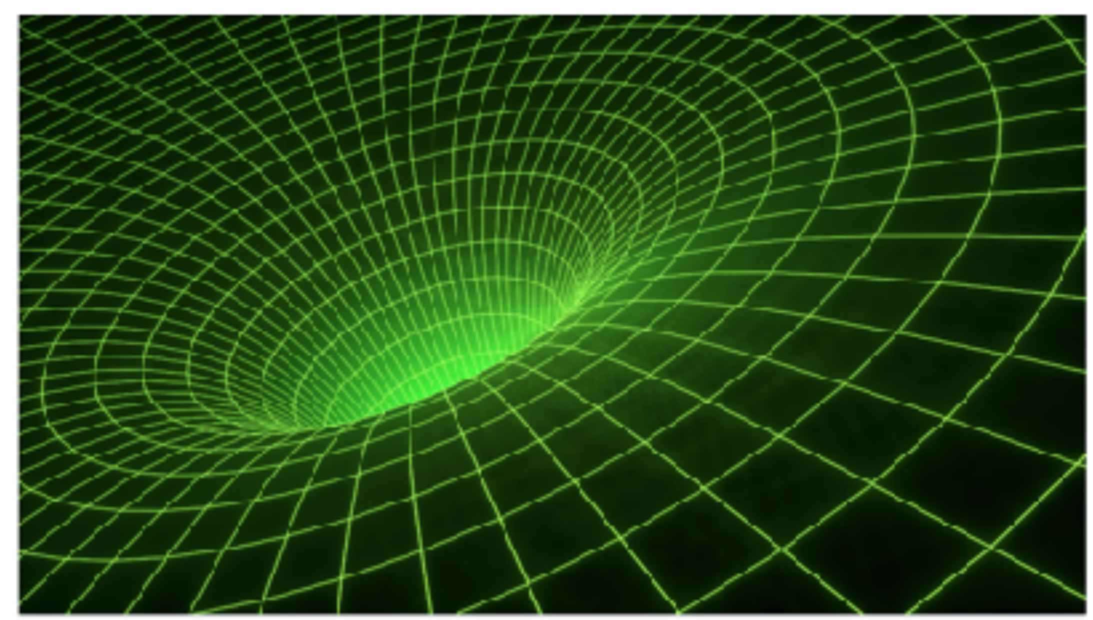
Weak

Dominant

Null

These play a key role in our basic understanding of the universe:

Singularity theorems, no traversable wormholes, no time machines, etc.



	Is it true?	Violated by
Strong	not even close	free scalar
Weak	no	negative CC
Dominant	no	negative CC
Null	not quite	quantum effects

They are all violated in quantum field theory.

[Epstein, Glaser, Jaffe '65]

QFTs **do** satisfy

- Non-local energy conditions
- Energy-entropy bounds

This talk: Progress on understanding these bounds in QFT
(in flat spacetime, without gravity)

Motivation

- Fundamental properties of unitary QFT
- Impose constraints on couplings constants, anomaly coefficients
- Predictions for real-world systems, *e.g.* 3d Ising
- Connect information-theoretic approaches to QFT with more traditional observables, i.e. correlation functions
- Tractable limit of AdS/CFT: calculate everything on both sides

Focus on: null energy T_{uu}

Outline

I. The ANEC

II. Holographic CFTs

III. Energy vs. entropy

The ANEC: Averaged null energy condition

$$\int_\gamma du T_{uu} \geq 0$$

Null coordinates $u, v = t \pm x$

$$\gamma \;=\; \text{null ray}$$

Some background:
80's and 90's

- ANEC is sufficient for many of the GR theorems

e.g. [Borde '87]

- Derived in various free theories by explicit calculation of $T_{\mu\nu}$

[Klinkhammer '91; Wald, Yurtsever '91; Folacci '92; Ford, Roman '95]

2000's: Conformal Collider Bounds

[Hofman and Maldacena '08]

Evaluate the ANEC in a state created by the stress tensor itself,

$$\langle T(p)| \int du T_{uu} |T(p)\rangle \geq 0$$

Constrains the OPE coefficients C_{TTT}

$$4d: \quad \frac{1}{3} \leq \frac{a}{c} \leq \frac{31}{18}$$

Holographic dual:

$$S = \int \sqrt{g} \left(R - 2\Lambda + \alpha R^2 + \cdots \right)$$

Einstein gravity has *a*=*c*; so this constrains higher curvature couplings

Same constraints follow from bulk causality

[Brigante, Liu, Myers, Shenker, Yaida '07]
[Hofman '09]

But is the ANEC true in interacting QFTs?

Or an extra constraint we should impose?

Answer: The ANEC is a consequence of unitarity.

Recently, three derivations in interacting QFT in $d > 2$:

- Holographic
- Relative entropy **[postpone…]**
- Causality of correlation functions

ANEC from Causality

Consider a 4-point correlation function

$$G = \langle \psi(x_3)\mathcal{O}(x_1)\mathcal{O}(x_2)\psi(x_4)\rangle$$

Unitarity + Causality + Boostrap methods

➡ Conformal collider bounds

[TH, Jain, Kundu ’15, ’16]
[Komargodski, Kulaxizi, Parnachev, Zhiboedov ’16]
[Hofman, Li, Meltzer, Poland, Rejon-Barrera '16]

➡ and the full ANEC

[TH, Kundu, Tajdini '16]

Ingredients:

(1) OPE in the lightcone limit gives

$$\mathcal{O}\mathcal{O} \sim 1 + \int_{-\infty}^{\infty} du T_{uu}$$

This relates the ANEC to properties of 4-point functions,

$$G \sim 1 + \langle\psi| \int_{-\infty}^{\infty} du T_{uu} |\psi\rangle$$

(2) Causality:

$$[\mathcal{O}(x_1), \mathcal{O}(x_2)] = 0 \qquad \text{(spacelike)}$$

tells us the domain where *G* is analytic. $\oint G = 0$

[TH, Kundu, Tajdini '16]

This leads to a sum rule for the integrated null energy:

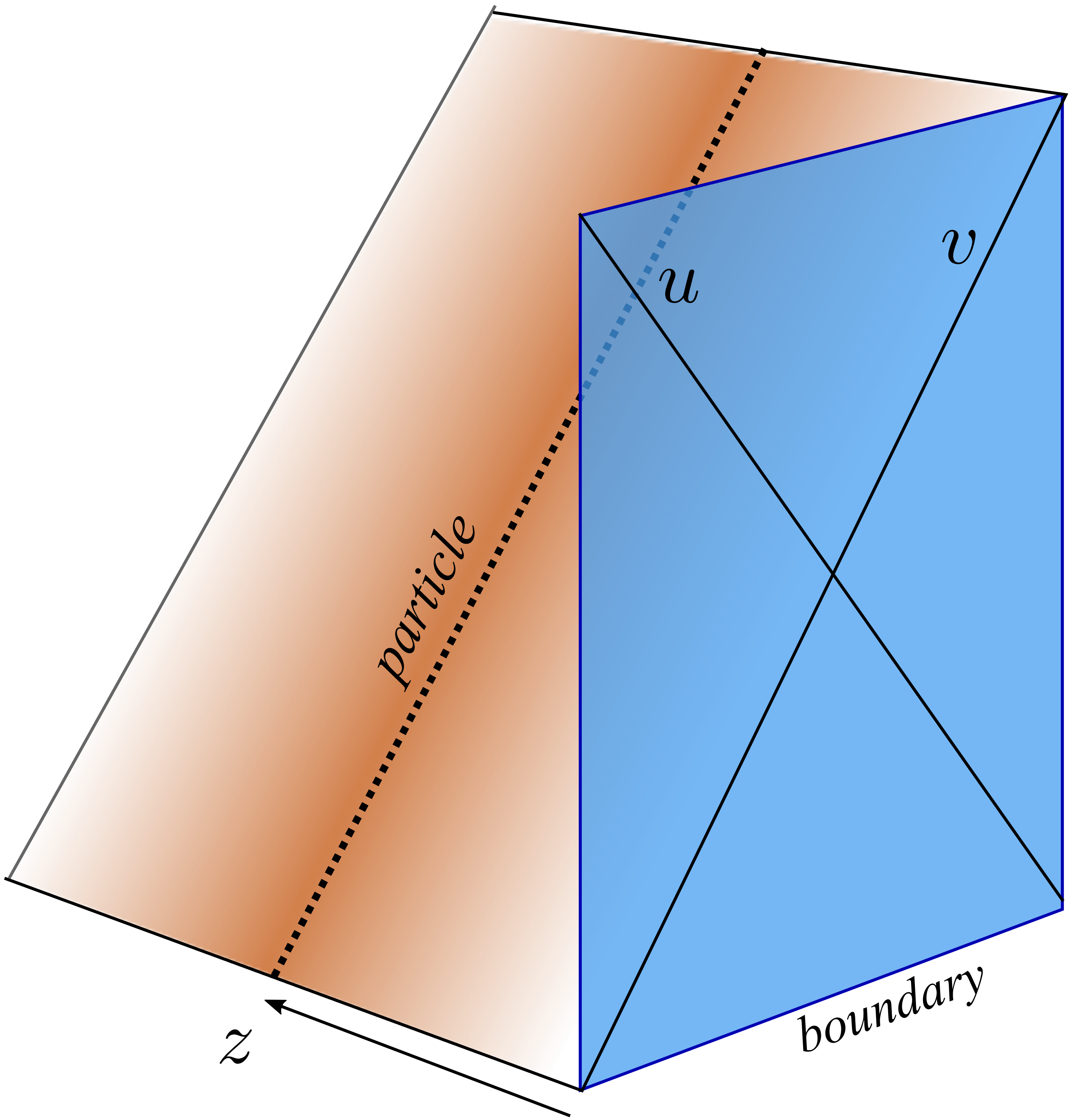
$$\boxed{\begin{gathered}\langle\psi|\int_{-\infty}^{\infty} du T_{uu}|\psi\rangle = \int \langle[\psi,\mathcal{O}][\psi,\mathcal{O}]\rangle \\ \geq 0\end{gathered}}$$

(3) The "double discontinuity" on the right is positive by unitarity. This implies the ANEC.

compare: Lorentzian OPE inversion formula [Caron-Huot '17]

ANEC from Holography

[Kelly, Wall '14]



The particle stays at fixed z close to the AdS boundary.

Time delay:

$$\Delta v \sim \int_{-\infty}^{\infty} du h_{uu}$$
$$\sim \int_{-\infty}^{\infty} du \langle T_{uu} \rangle$$
$$\geq 0$$

This proves the ANEC in holographic CFTs (large N, etc.)

But clearly the logic is very similar to the general CFT argument, which did not require large N.

Both use causality in the lightcone limit

Applications and Extensions of the ANEC

Applications and Extensions of the ANEC

1) The Higher Spin ANEC

[TH, Kundu, Tajdini '16]
see also:
[Komargodski, Kulaxizi, Parnachev, Zhiboedov '16]

Positive sum rule for integrated higher spin operators (J even)

$$\langle\psi|\int_{-\infty}^{\infty} X_{uuuu\cdots}|\psi\rangle = \int dz\, z^{J-2}\langle[\psi,\mathcal{O}][\psi,\mathcal{O}]\rangle$$

$$\geq 0$$

This is an experimental prediction for systems in the lab— *e.g.* 3d Ising or the O(2) model

Confirmed by numerical bootstrap in 3d Ising for $J \leq 40$

[Simmons-Duffin '16]

Applications and Extensions of the ANEC

2) The Continuous Spin ANEC

[Kravchuk, Simmons-Duffin '18]

What if J is not an integer?

$$\langle\psi|\ ?\ |\psi\rangle = \int dz|z|^{J-2}\langle[\psi,\mathcal{O}][\psi,\mathcal{O}]\rangle$$

The right-hand side is still positive.

But does this correspond to some positive operator on the left?

Yes: a nonlocal "light ray operator" with non-integer spin

$$\int du du' X(u,u')$$

These operators naturally appear in the OPE data upon analytic continuation in spin, and control high-energy (Regge) scattering.

Applications and Extensions of the ANEC

3) Interference in the Conformal Collider

Evaluate the ANEC in superpositions

$$|\Psi\rangle = \alpha T(p)|0\rangle + \beta \mathcal{O}(p)|0\rangle$$

$$\begin{pmatrix} c_{TTT} & c_{TTO} \\ c_{TTO} & c_{OTO} \end{pmatrix} \geq 0$$

Constrains the off-diagonal (*TTO*) couplings by the diagonal (*TTT*, *OTO*) couplings:

$$\sum_i f_i |c_{TTO_i}|^2 \leq c_{TTT}$$

Eigenvalue repulsion makes the ANEC stronger

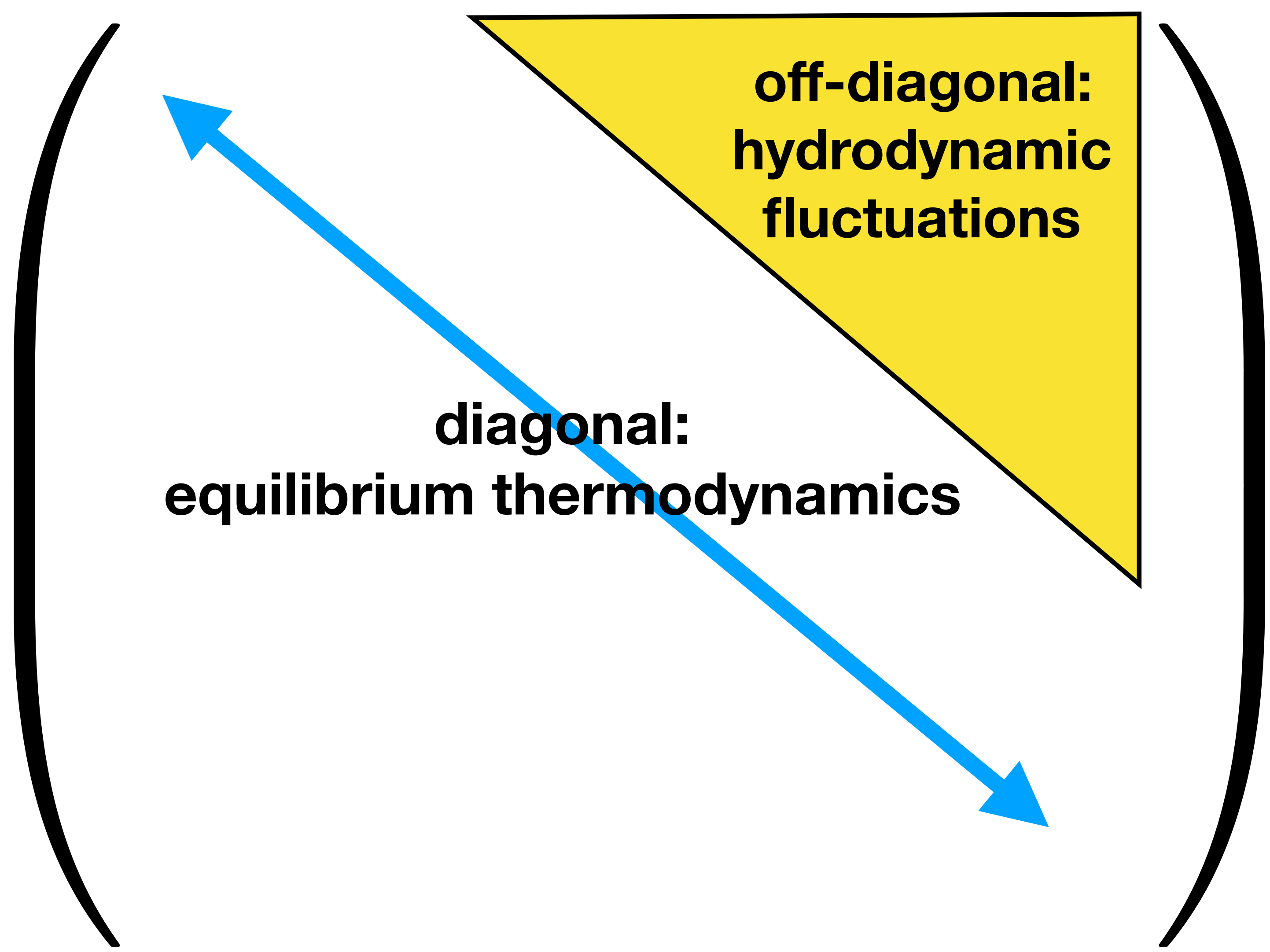
[Cordova, Maldacena, Turiaci '17]

Applications and Extensions of the ANEC

4) Bounds on Transport

Apply ANEC to all states in the microcanoncal ensemble at energy *E*

$$\langle i| \int du T_{uu} |j\rangle \sim$$



$$\geq 0$$

[Delacrétaz, TH, Hartnoll, Lewkowycz '18]

Applications and Extensions of the ANEC

Constraints on transport coefficients vs. thermalization length

$$\Lambda_{hydro} \lesssim f\left(\frac{\eta}{s}, \frac{\zeta}{s}, c_{sound}, \cdots\right)$$

[Delacrétaz, TH, Hartnoll, Lewkowycz '18]

Other (non-ANEC) bounds

Upper bounds on transport coefficients also imposed by causality of hydrodynamic 2-point functions.

[Baier, Romatschke, Son, Starinets, Stephanov '07]
[Romatschke '09]
[TH, Hartnoll, Mahajan '17]

These also have been derived rigorously in non-relativistic QM.

[Han, Hartnoll '18]

Quark-gluon plasma?

Applications and Extensions of the ANEC

5) New Unitarity Bounds in 4d [Cordova, Diab '17]

For $(k, \bar{k})$ Lorentz reps, the ANEC gives

$$\Delta \geq \max(k, \bar{k})$$

(partly conjecture)

~2x stronger for chiral reps!

6) Parity-violating OPE coefficients [Chowdhury, David, Prakash '17]

$$3d : \quad \langle TTT \rangle, \quad \langle JJT \rangle$$

Saturated by large-*N* Chern-Simons-Matter theories

Outline

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II. Holographic CFTs

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The goal is to show that energy conditions become much stronger in large-*N*, holographic theories, and that this can help us understand the emergence of gravity in these theories.

[Maldacena, Shenker, Stanford '15]

First some comments on the chaos bound:

The chaos bound constrains thermal 4-point functions (OTOCs),

$$G \sim \langle \psi(0)\mathcal{O}(t)\psi(0)\mathcal{O}(t)\rangle_\beta$$
$$\sim 1 - \frac{1}{N}e^{\lambda t} + \cdots$$

sign constraint

maximal Lyapunov $\lambda \leq \frac{2\pi}{\beta}$

This also constrains vacuum correlators, because they can be viewed thermal correlators in Rindler

Regge

Chaos

Dominated by high-spin exchanges

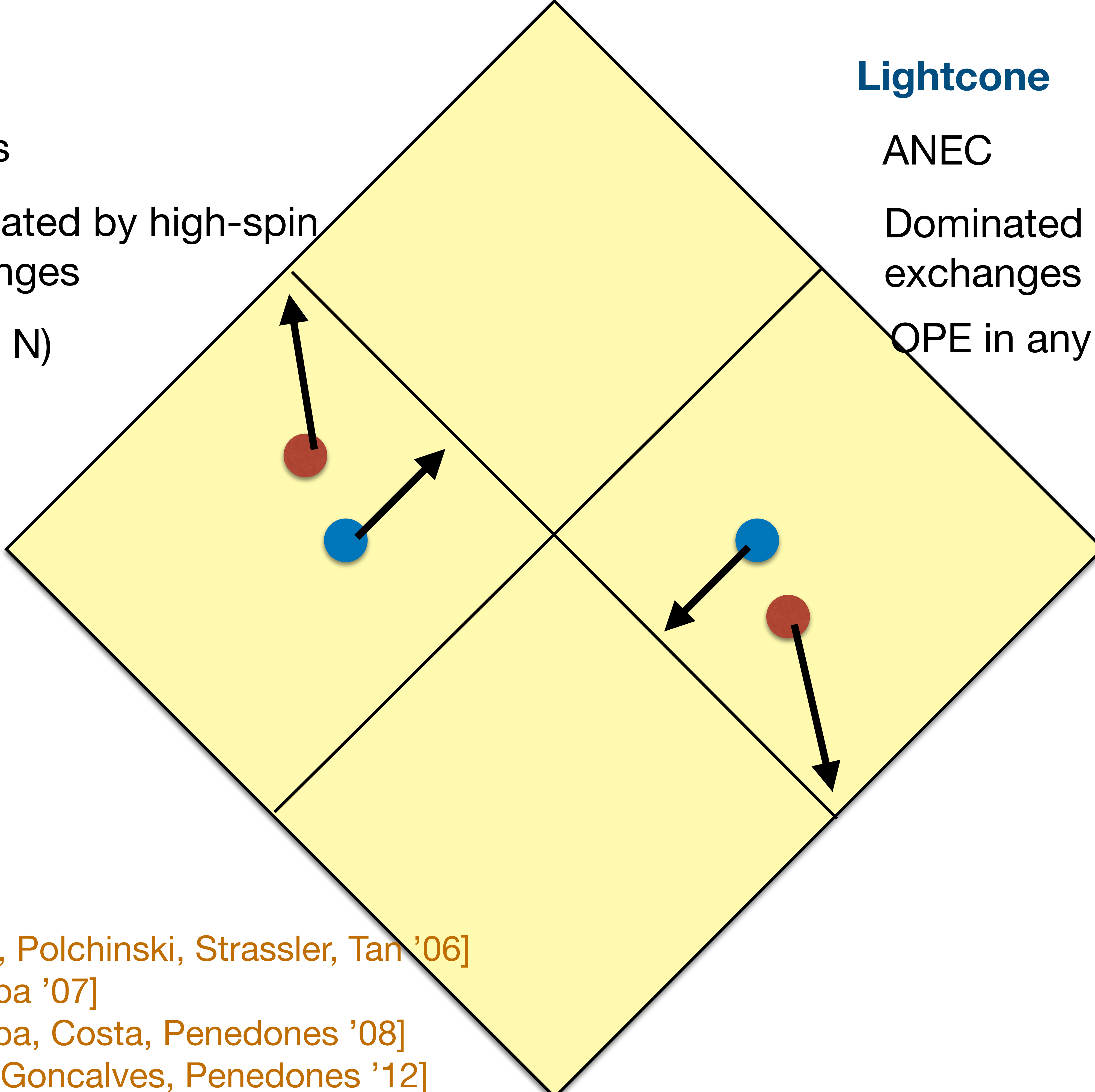
(Large N)

Lightcone

ANEC

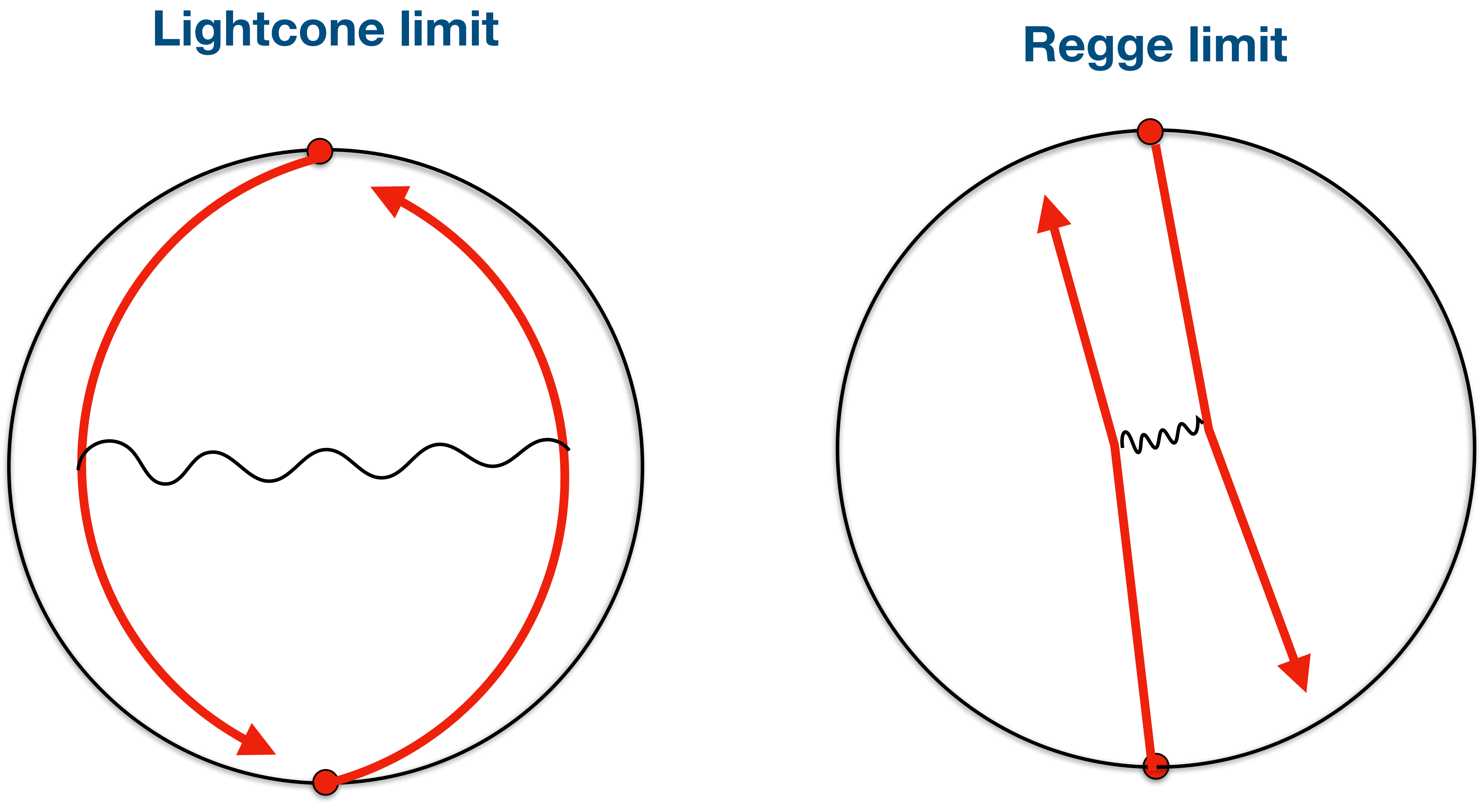
Dominated by low-twist exchanges

OPE in any CFT



[Brower, Polchinski, Strassler, Tan ’06]
[Cornalba ’07]
[Cornalba, Costa, Penedones ’08]
[Costa, Goncalves, Penedones ’12]

Two limits in the bulk of AdS:



We will now venture into the Regge regime. This requires large-N.

The goal is to derive aspects of local physics in AdS, directly from CFT.

Can we derive gravity in AdS directly from CFT?

In *2d*, often “yes” using Virasoro. But in higher dimensions?

(Set $d=4$).

At low energies, we expect roughly

Large *#d.o.f.*   +   Large gap   —>   emergent gravity

specific conjecture in:
[Heemskerk, Polchinski, Penedones, Sully]

But what exactly do you need to show?

Any theory of quantum gravity looks like ***Einstein*** gravity at low energies,

$$S = \frac{1}{G_N} \int \left(-2\Lambda + R + \frac{c_2}{M^2} R^2 + \cdots \right)$$

e.g. 5d Gauss-Bonnet

With *M* = string mass and $c_2 \sim O(1)$

[Camanho, Edelstein, Maldacena, Zhiboedov '14]

To "derive AdS gravity from CFT" means to show that all consistent CFTs (in some class) correspond to bulk theories with higher derivative terms suppressed by the string scale.

This suppression is the hallmark of a local bulk.

Ultimately non-perturbative; but for now, correlators in vacuum

A big piece of this was proved by bootstrap in:

[Heemskerk, Polchinski, Penedones, Sully ’09]
[Penedones ’10]
[Fitpatrick, Kaplan, Penedones, Raju, van Rees ’11]
etc.

Lagrangians in AdS $\Longleftrightarrow$ Solutions of CFT crossing equation order by order in *1/N*

But this leaves unanswered:

Why *Einstein* gravity in the bulk?

The simplest version of this question is for 3-point functions:

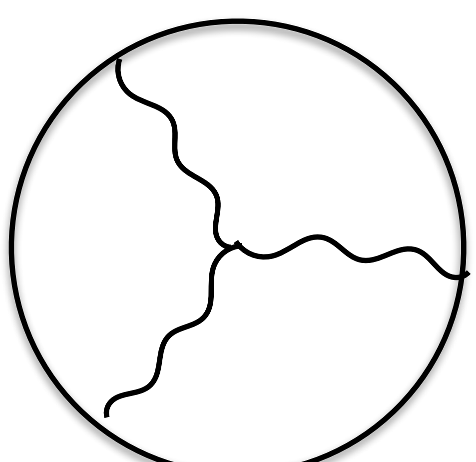
Why does some class of CFTs have universal <TTT> correlators?

In a general CFT, there are 3 free coefficients:

$$\langle TTT\rangle_{cft} = a\langle TTT\rangle_1 + c\langle TTT\rangle_2 + t_4\langle TTT\rangle_3$$

But Einstein gravity is very special:

$$\langle TTT\rangle =$$



$$a = c \quad \text{and} \quad t_4 = 0$$

Similar questions apply to matter couplings:

Why is $\langle TT\mathcal{O}\rangle$ universal?

Bulk:

$$S \sim \cdots + \int \sqrt{g}\phi C^2_{\mu\nu\alpha\beta}$$

So effective field theory in the bulk predicts

$$\langle TT\mathcal{O}\rangle \sim 0$$

Or more accurately,

$$\langle TT\mathcal{O}\rangle \sim \frac{1}{\Delta_{gap}^{\#}}$$

These universal 3-point functions have now been derived from CFT.

The basic idea:

Unitarity + Causality + Bootstrap methods

➡ ANEC in the lightcone limit [first part of talk]

➡ universality of spinning 3-point functions in the Regge limit (in large-*N* theories with a gap)

[Afkhami-Jeddi, TH, Kundu, Tajdini ’16, ’17]
[Li, Meltzer, Poland ’17]
[Kulaxizi, Parnachev, Zhiboedov ’17]
[Costa, Hansen, Penedones ’17]
[Meltzer, Perlmutter ’17]
[Afkhami-Jeddi, Kundu, Tajdini ’18]

These papers use several different methods, looking at different parts of the amplitude.

Conclusion: The ANEC gets replaced by a stronger condition in holographic CFTs.

This stronger condition interpolates from the Hofman-Maldacena constraints

$$\frac{1}{3} \leq \frac{a}{c} \leq \frac{31}{18}$$

to the Einstein gravity result,

$$1 \leq \frac{a}{c} \leq 1 \qquad\qquad i.e. \;\; a = c$$

(Several constraints from different polarizations.)

Universality of <*TTT*> and <*TTO*>

Outline

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Bekenstein Bound [Bekenstein '81]

$$S \leq 2\pi RE$$

Reformulated as relative entropy:

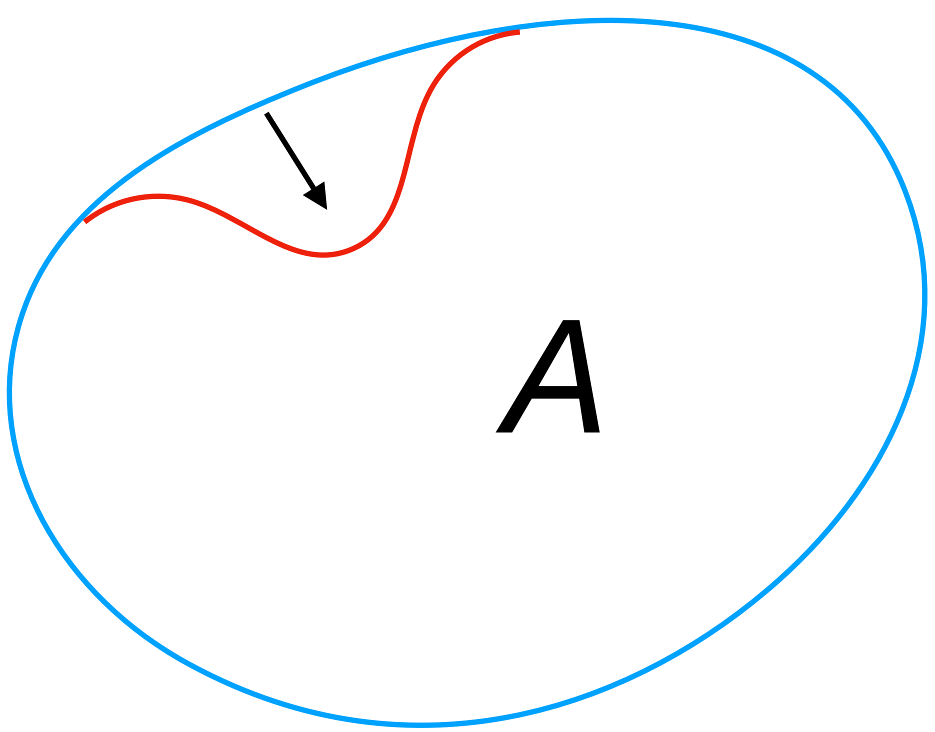
[Marolf, Minic, Ross '03]
[Marolf '04]
[Casini '08]

$$S_{rel}(\rho|\rho_{vacuum}) = \Delta\langle H\rangle - \Delta S \geq 0$$

Measures distinguishability from vacuum.

Monotonicity

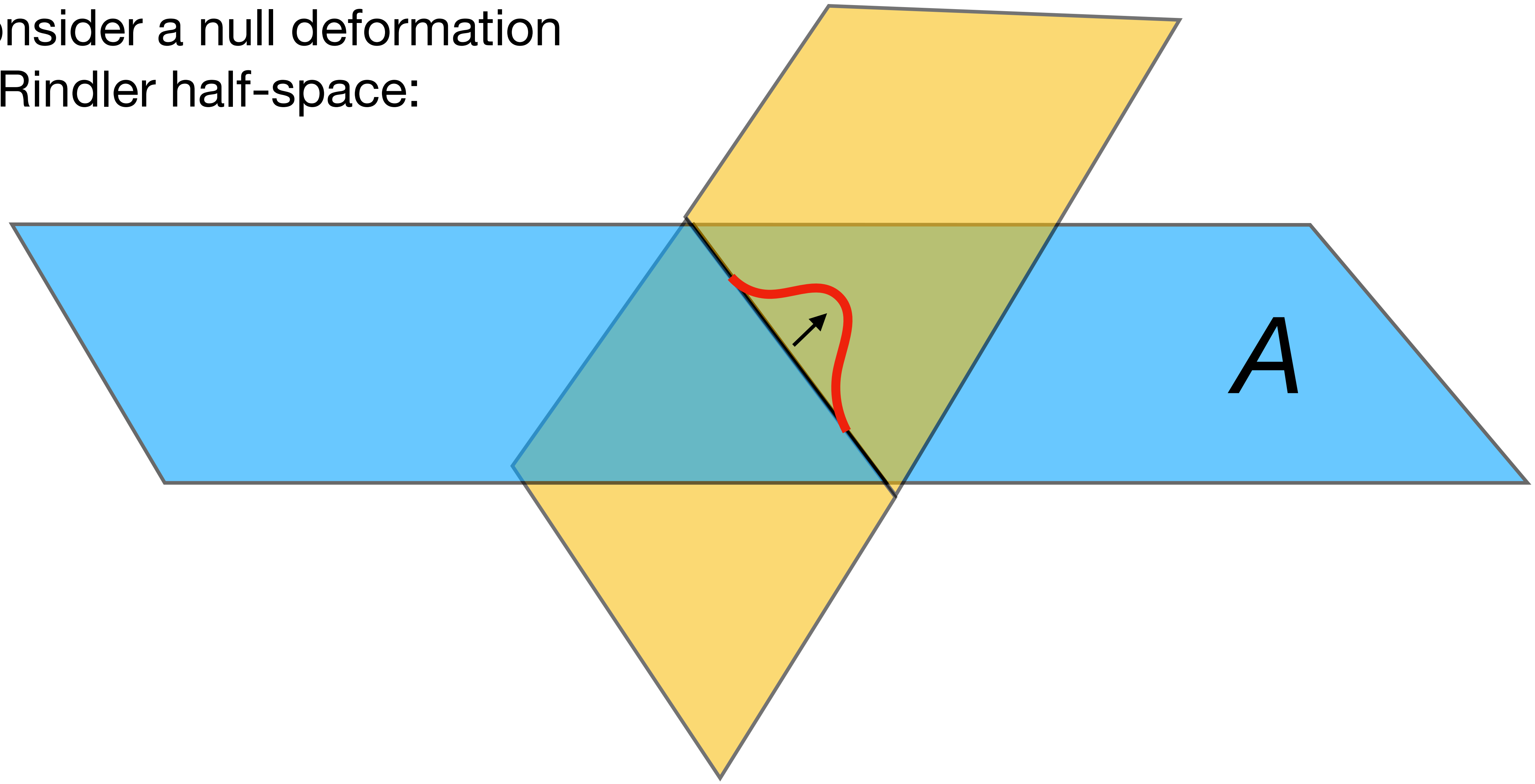
Relative entropy decreases under deformations that "shrink" the region



$$\frac{d}{d\lambda} S_{rel} \leq 0$$

"'harder to distinguish"

Consider a null deformation
of Rindler half-space:



For this deformation, monotonicity gives the “half-ANEC”:

$$-\frac{d}{d\lambda}S_{rel} = \int_0^\infty du T_{uu} + \frac{1}{2\pi}S'_A \geq 0$$

Adding the same formula for the complement region gives the regular ANEC. (Entanglement terms cancel.)

[Faulkner, Leigh, Parrikar, Wang ’16]
see also: [Wall ’10], [Blanco, Casini ’13]

Relative entropy is *positive* and *monotonic* in any quantum system.

A surprise: In QFT, there is a stronger condition,

$$\frac{d^2}{d\lambda^2} S_{rel} \geq 0$$

Since $\frac{d}{d\lambda} S_{rel} \to 0$ at infinity, this implies monotonicity

This parallels the derivation of the second law of black hole thermodynamics:

$$\text{focusing} \quad \Rightarrow \quad Area'' \leq 0 \quad \Rightarrow \quad Area' \geq 0$$

[Bousso, Fisher, Leichenauer, Wall '15]
[Bousso, Fisher, Leichenauer, Koeller, Wall '15]
[Balakrishnan, Faulkner, Khandker, Wang '17]

The 2nd derivative has local and non-local terms along the horizon.

The non-local terms are positive by strong subadditivity.

The local term is a new *local energy condition:*

The *QNEC*: Quantum Null Energy Condition

$$\langle T_{uu} \rangle \geq \frac{1}{2\pi} S_A''$$

Recall the theorem: “no local positive energy in QFT”

This evades the theorem because the r.h.s. is not an operator.

[Bousso, Fisher, Leichenauer, Wall ’15]

Status of the QNEC

- Motivated by coupling to gravity and asking for a quantum analogue of the focusing equation; the QNEC survives as Newton's constant —> 0

  [Bousso, Fisher, Leichenauer, Wall '15]

- Derived in free QFT

  [Bousso, Fisher, Leichenauer, Koeller, Wall '15]

- Derived in holographic theories from a *local* causality condition
  (cf. ANEC and a=c from boundary causality)

  [Koeller, Leichenauer '15]

- Derived in interacting QFT

  [Balakrishnan, Faulkner, Khandker, Wang '17]

- Found to be *saturated* in interacting theories!

$$\langle T_{uu} \rangle = \frac{1}{2\pi} S''_A$$

[Ecker, Grumiller, van der Schee, Stanzer '17]
[Leichenauer, Levine, Shahbazi-Moghaddam '18]
[Khandker, Kundu, Li '18]
[Balakrishnan et al, work in progress]

QNEC vs ANEC

Integrating gives the ANEC,

$$\int_{-\infty}^{\infty} du T_{uu} = \int_{-\infty}^{\infty} d\lambda \frac{d^2 S_{rel}}{d\lambda^2}$$
$$= \int_{-\infty}^{\infty} d\lambda \left(SSA + \cancel{QNEC}\right)$$

If the QNEC is saturated, the integrand comes entirely from the nonlocal terms. Therefore

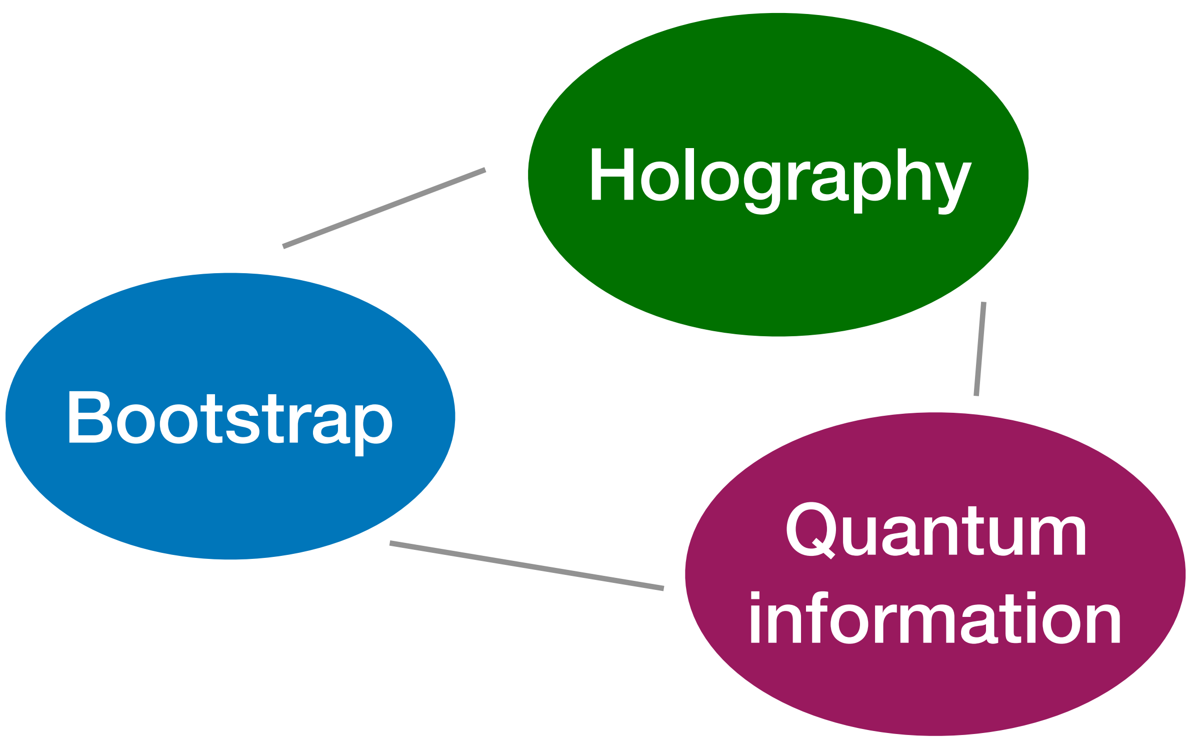
Strong subadditivity of a “skinny” region $\Longleftrightarrow$ ANEC

And the proof of the QNEC by [Balakrishnan, Faulkner, Khandker, Wang] becomes a field-theoretic proof of SSA.

[Leichenauer, Levine, Shahbazi-Moghaddam ’18]

Conclusions

- ANEC <— Causality in lightcone limit
- Universal 3-point functions @ large *N* <— Causality in Regge limit
- QNEC <— Local causality in the bulk



These are fundamental properties of QFT.

But each was first discovered by coupling the QFT to gravity, or from holography.