Eternal traversable worm hole in 2D

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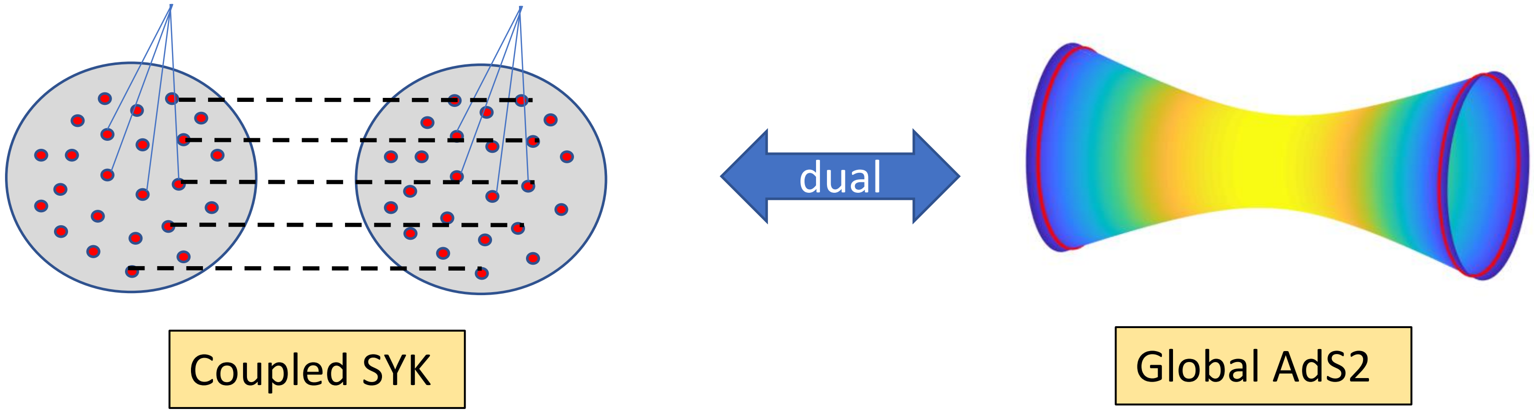
Ref: J. Maldacena & XLQ, arxiv: 1804.00491

Goal:



Outline:

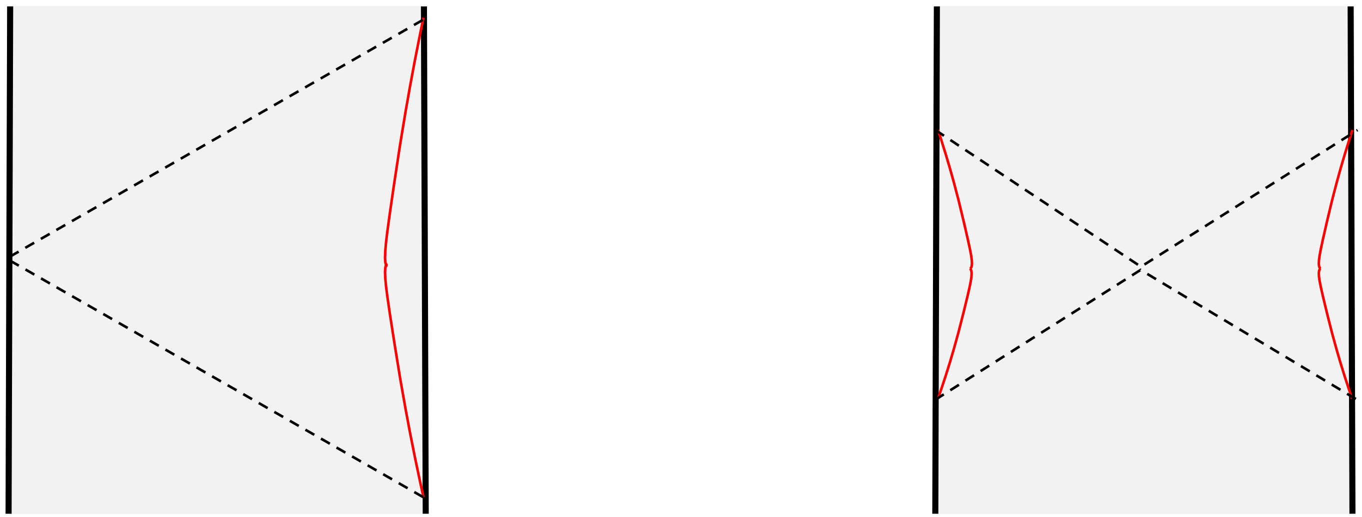
1. Overview of the AdS2-SYK duality
2. Proposal: Global AdS2 ⇔ coupled SYK model
3. Low energy effective theory
4. Beyond low energy: large q limit
5. Finite temperature: Hawking-Page type phase transition

Jackiw-Teitelboim gravity

$$S = \frac{\phi_0}{2}\left[\int R + 2\int_{\text{Bdy}} K\right] + \frac{1}{2}\left[\int \phi(R+2) + 2\phi_b \int_{\text{Bdy}} K\right] + S_{\text{matter}}[\chi, g]$$

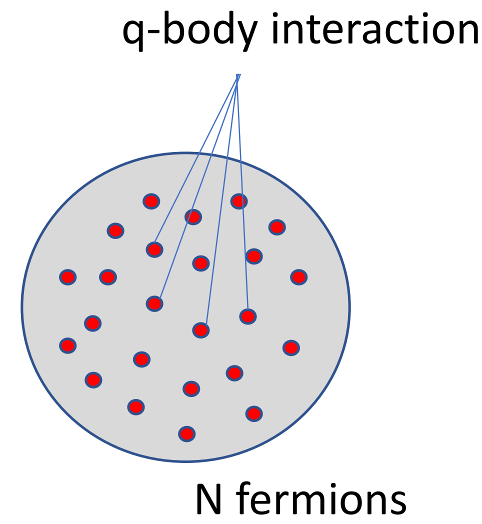
Jackiw '85, Teitelboim '83

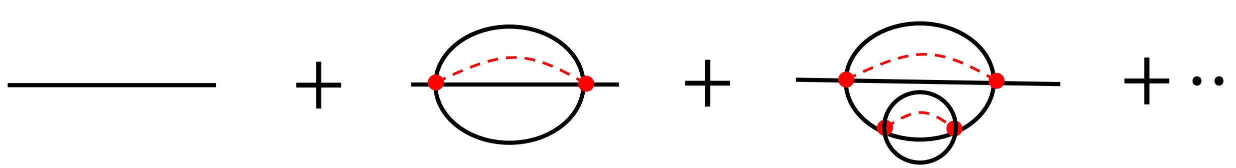
- No bulk graviton.
- Boundary condition $\phi = \phi_b$
- Reduction to boundary dynamics $S = -\phi_r \int \{t_P(u), u\} du$
- Different solutions



Sachdev-Ye-Kitaev model

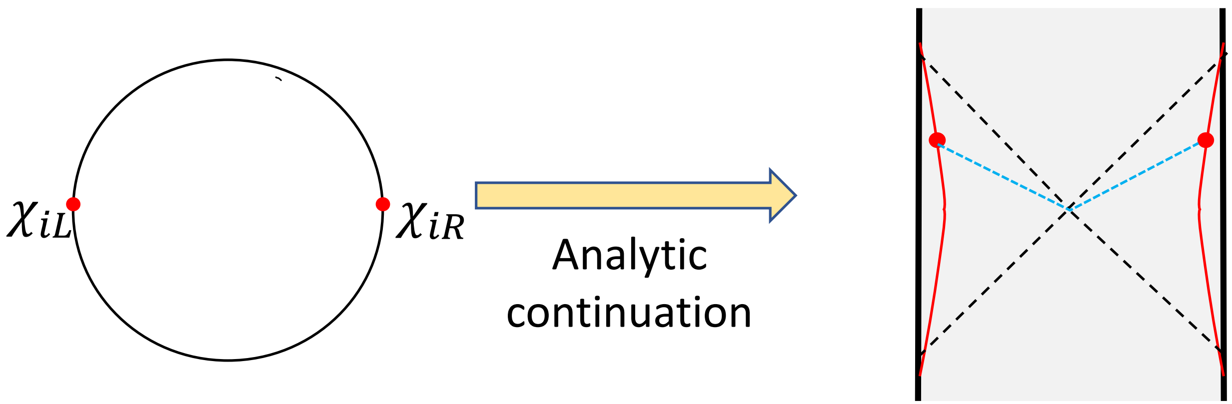
- $H = \sum_{ijkl} J_{ijkl} \chi_i \chi_j \chi_k \chi_l$ with Gaussian random coupling J_{ijkl}.
  (Sachdev-Ye, '93, Kitaev '15)
- Or complex fermion model
  $H = \sum_{ij,kl} J_{ijkl} c_i^{+} c_j^{+} c_k c_l$.
- Generalization (Maldacena-Stanford '16) :
  $H = \sum_{i_1 i_2 \dots i_q} J_{i_1 i_2 \dots i_q} \chi_{i_1} \chi_{i_2} \cdots \chi_{i_q}$
- Averaging over disorder
- $\overline{Z^n} \simeq \overline{Z}^{\mathrm{n}}$ in large N limit.
- Large N order parameter
- $G(\tau_1, \tau_2) = \frac{1}{N} \sum_i \langle \chi_i(\tau_1) \chi_i(\tau_2) \rangle =$



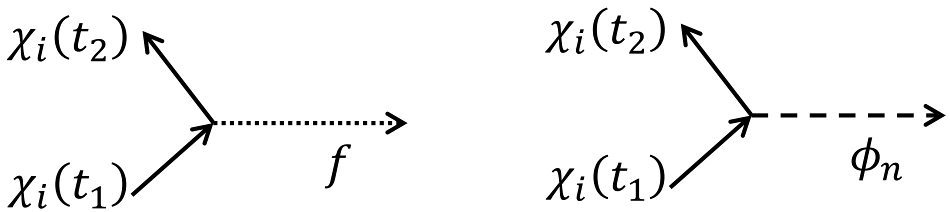


Sachdev-Ye-Kitaev model

- Approximate conformal invariance at low energy
- $G(\tau_1, \tau_2) \to G_f = \left(f'(\tau_1) f'(\tau_2)\right)^{\Delta} G\left(f(\tau_1), f(\tau_2)\right)$
- Low energy manifold $\mathrm{Diff}(S^1)/SL(2,R)$
- Effective action $S_{eff}[G_f] = -\alpha \int d\tau \, \mathrm{Sch}\{\tan f(\tau), \tau\}, \alpha \propto \frac{N}{J}$
- Thermofield double dual to AdS2 black hole for $\frac{1}{N} \ll \frac{1}{\beta J} \ll 1$



- An infinite tower of massive matter fields in the bulk

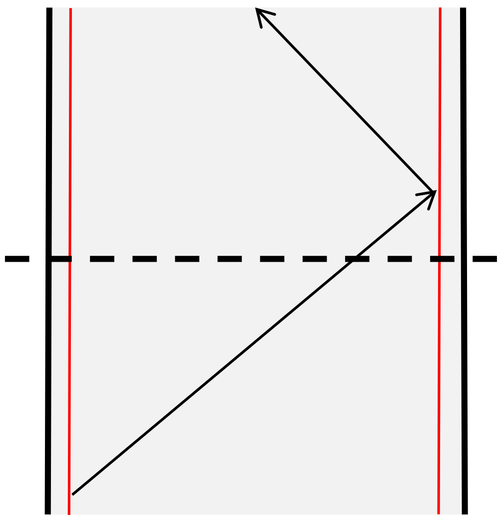


Goal: finding a dual theory of global AdS2

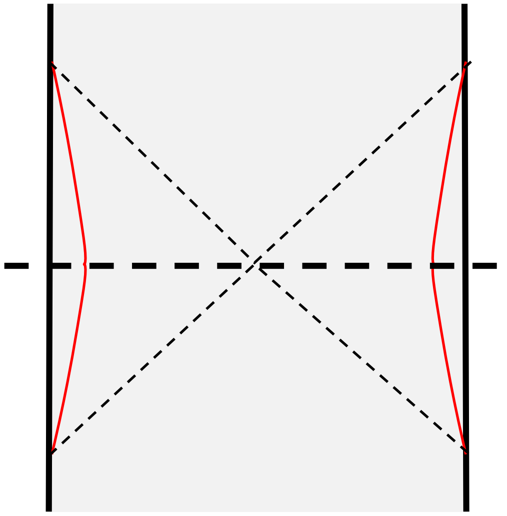
- Two causally connected boundaries
- Requires negative averaged null energy

$$-2\phi_r = -\left(\sin^2\sigma\partial_+\phi\right)\Big|_{-\infty}^{+\infty} = \int_{-\infty}^{\infty} dX^+ T_{X^+X^+}$$

  from matter field
- An eternal version of traversable WH
  (Gao, Jafferis, Wall '16, Maldacena, Stanford, Yang '16)
- Basic idea: The same state as $|TFD(t=0)\rangle$ but with different time evolution.
- Finding dual of global AdS_2
  $\simeq$ Finding a Hamiltonian with $|TFD(t=0)\rangle$ as ground state

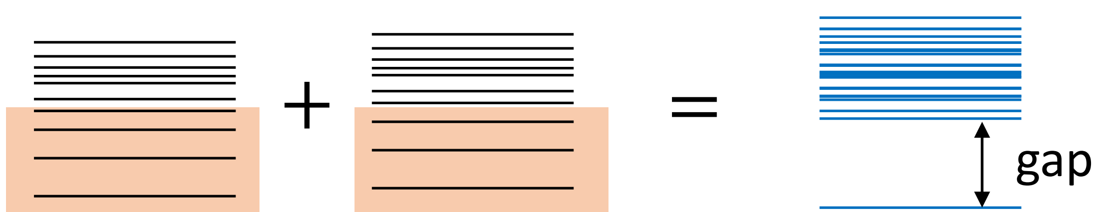


vs



Conjecture

- We conjecture that
$$SYK_L + SYK_R + relevant\ coupling$$
has the ground state $|\mathrm{G}\rangle \simeq |TFD\rangle$
- Simplest model
- $H = \sum J_{i_1 i_2 \dots i_q}\left(i^{\frac{q}{2}}\chi_{i_1 L}\chi_{i_2 L}\dots\chi_{i_q L} + i^{-\frac{q}{2}}\chi_{i_1 R}\chi_{i_2 R}\dots\chi_{i_q R}\right) + i\mu\sum_i \chi_{iL}\chi_{iR}$
- Intuition: small μ couples low energy states more strongly.
- More precise argument: Variationally minimize free energy among reparameterizations G_f
- Qualitatively similar phenomena in higher dimensional CFT (see XLQ, Katsura, Ludwig '12)

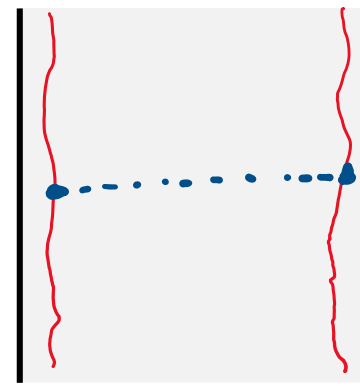


Low energy effective theory

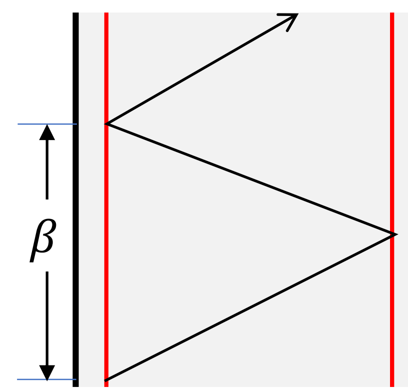
Repara. of $\langle \chi_{iL}(u)\chi_{iR}(u)\rangle$

- Coupled reparameterization modes

$$S = N\int du\left\{-\frac{\alpha_S}{\mathcal{J}}\left(\{\tan\frac{t_l(u)}{2},u\}+\{\tan\frac{t_r(u)}{2},u\}\right)+\mu\frac{c_\Delta}{(2\mathcal{J})^{2\Delta}}\left[\frac{t_l'(u)t_r'(u)}{\cos^2\frac{t_l(u)-t_r(u)}{2}}\right]^\Delta\right\}$$



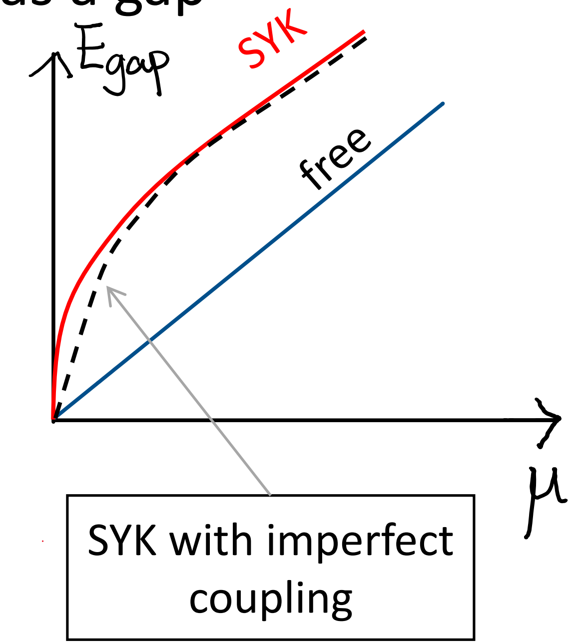
- Solution $t_l(u) = t_r(u) = \frac{2\pi}{\beta}u$,
  β^{-1}: "effective temperature". Also gap of the coupled model
- T determined by minimizing the free energy
  $\frac{F}{N} = -[\dots]\left(\frac{2\pi}{\beta}\right)^2 + [\dots]\mu\left(\frac{2\pi}{\beta}\right)^{2\Delta} \Rightarrow \frac{1}{\beta} \propto \mu^{\frac{1}{2-2\Delta}}$
- Two-point function periodic in time
  $\langle O(u_1)O(u_2)\rangle \propto \left(\frac{2\pi}{\beta}\frac{1}{\cos\left(\frac{\pi}{\beta}(u_1-u_2)\right)}\right)^\Delta.$



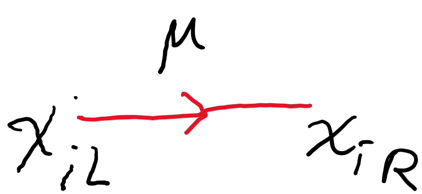
Physical interpretation on the boundary

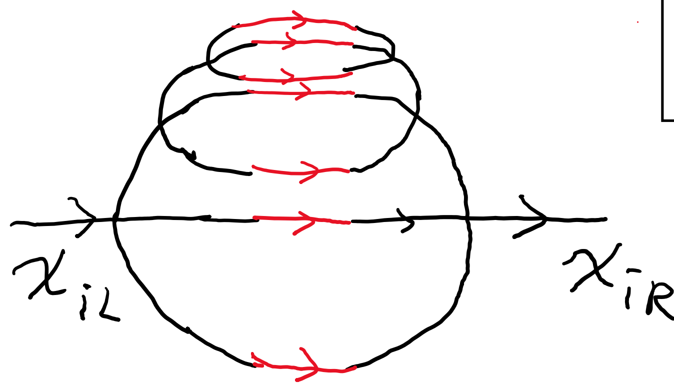
- Coupling μ is relevant. Coupled SYK model has a gap

$$E_{gap} \propto (\mu/J)^{\frac{1}{2-2\Delta}} J$$



- On comparison, for a free fermion
  $\mathrm{H} = \mu \sum_i i\chi_{iL}\chi_{iR}$, $E_{free} = 2\mu.$
- Since $\frac{1}{2-2\Delta} < 1$, $E_{gap} \gg E_{free}$ for small μ
- The SYK interaction terms help "tunneling" of fermion

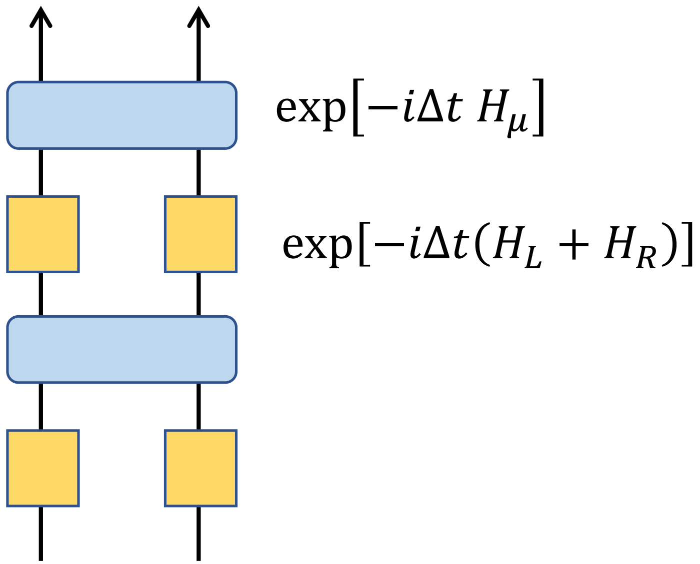


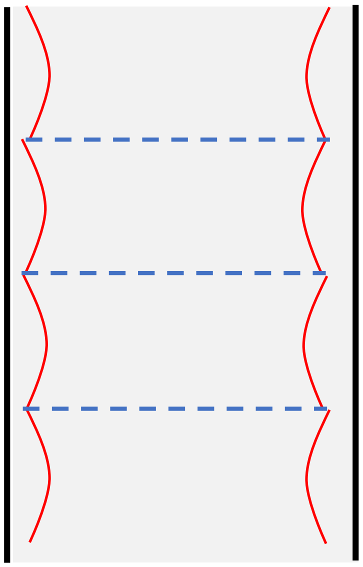


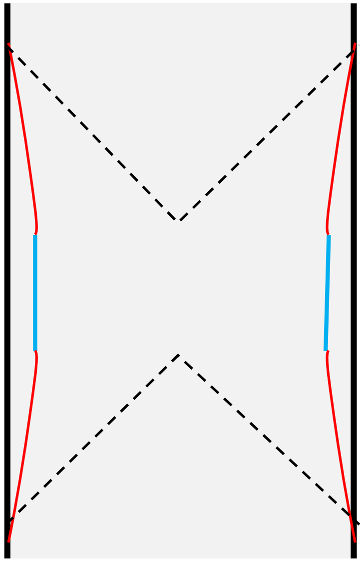
- This enhancement is only possible because identical J in L, R
- $E_{gap} \propto \mu$ if $\overline{J_L J_R} < \overline{J_L J_L}$ not perfect.

Relation to traversable worm hole

- Boundary point of view: μ term "undoes" the scrambling and restore the quantum computer to a clean state.



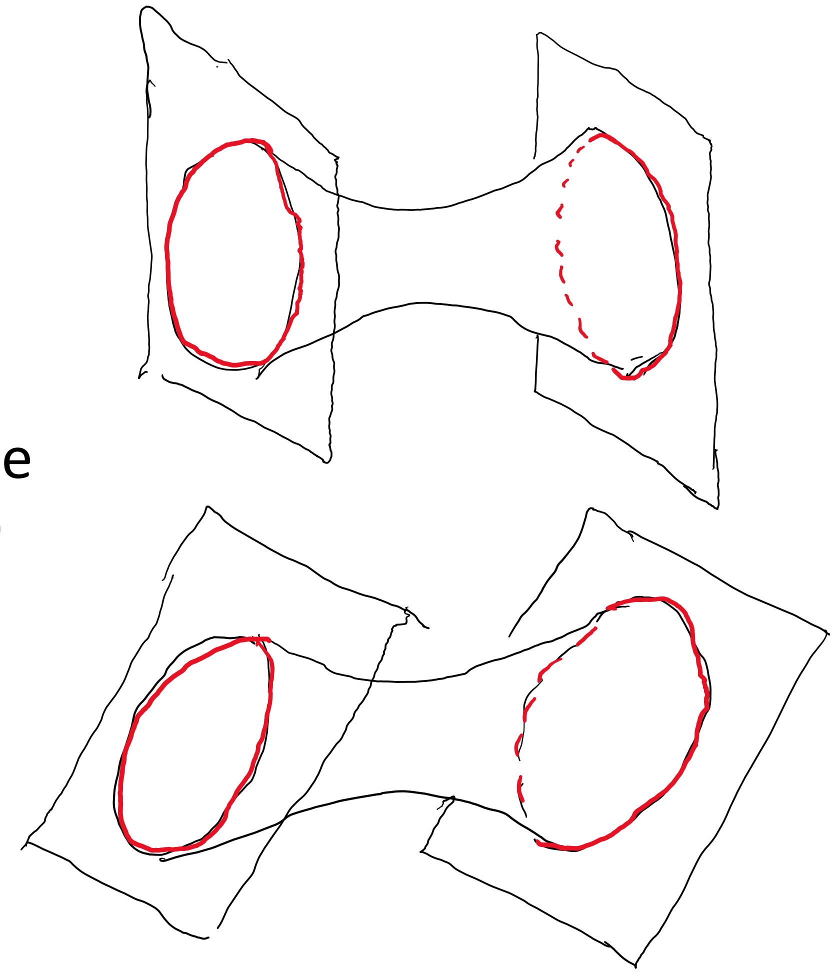




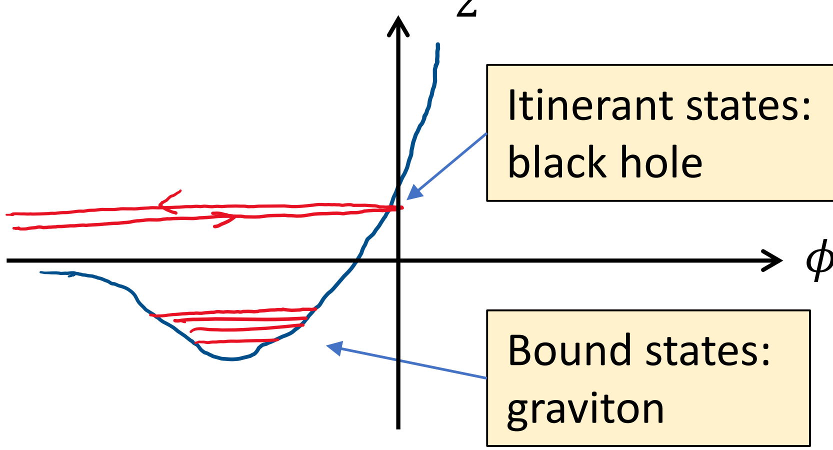
- By switching on a fine-tuned μ in the blackhole geometry, one can open a worm hole for arbitrarily long time. (compare with Gao, Jafferis, Wall '16, Maldacena, Stanford, Yang '16)

Low energy excitations

- More general solutions
- $SL(2,R)$ gauge symmetry
- $Q_0 = 0, Q_\pm = 0$
- General physical solutions are gauge equivalent to $t_l(u) = t_r(u) = \mathrm{t(u)}$
- $t'(u) = e^{\phi(u)}$
- $\phi'' = -e^{2\phi} + \Delta\eta e^{2\Delta\phi}$

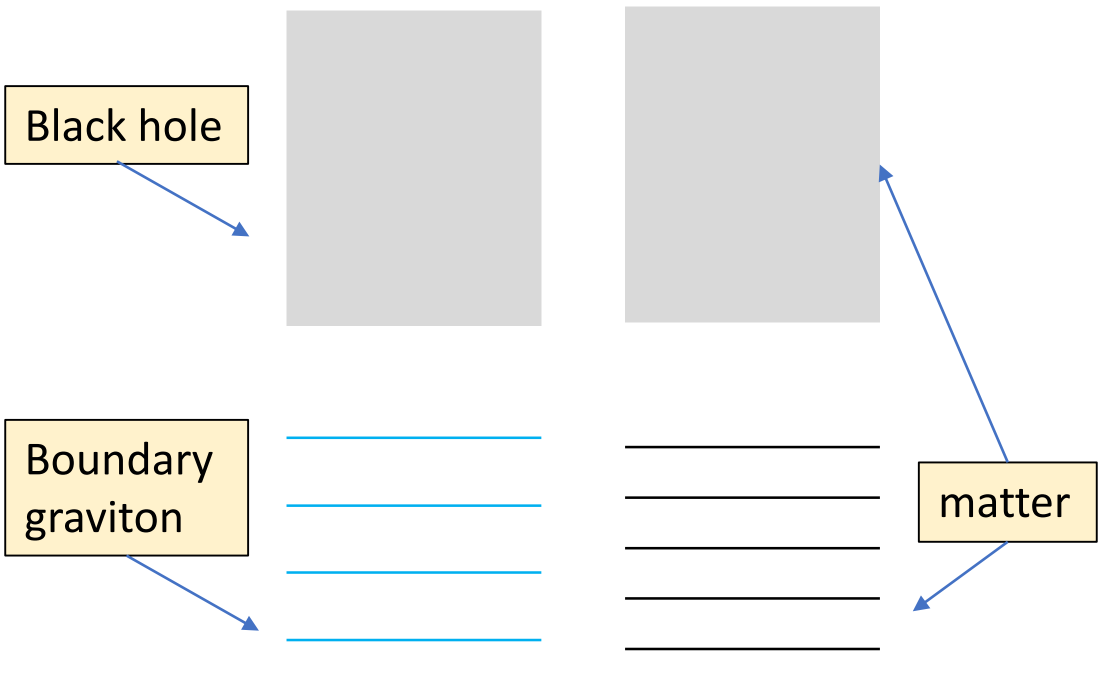


$$V(\phi) = \frac{1}{2}\left(e^{2\phi} - \eta e^{2\Delta\phi}\right)$$

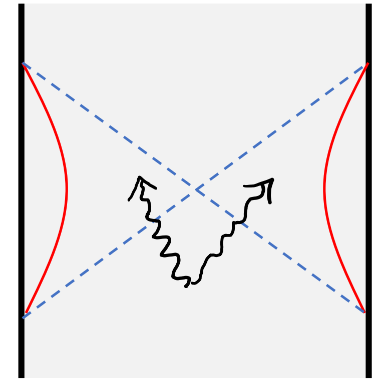


Low energy excitations

- Excitation spectrum

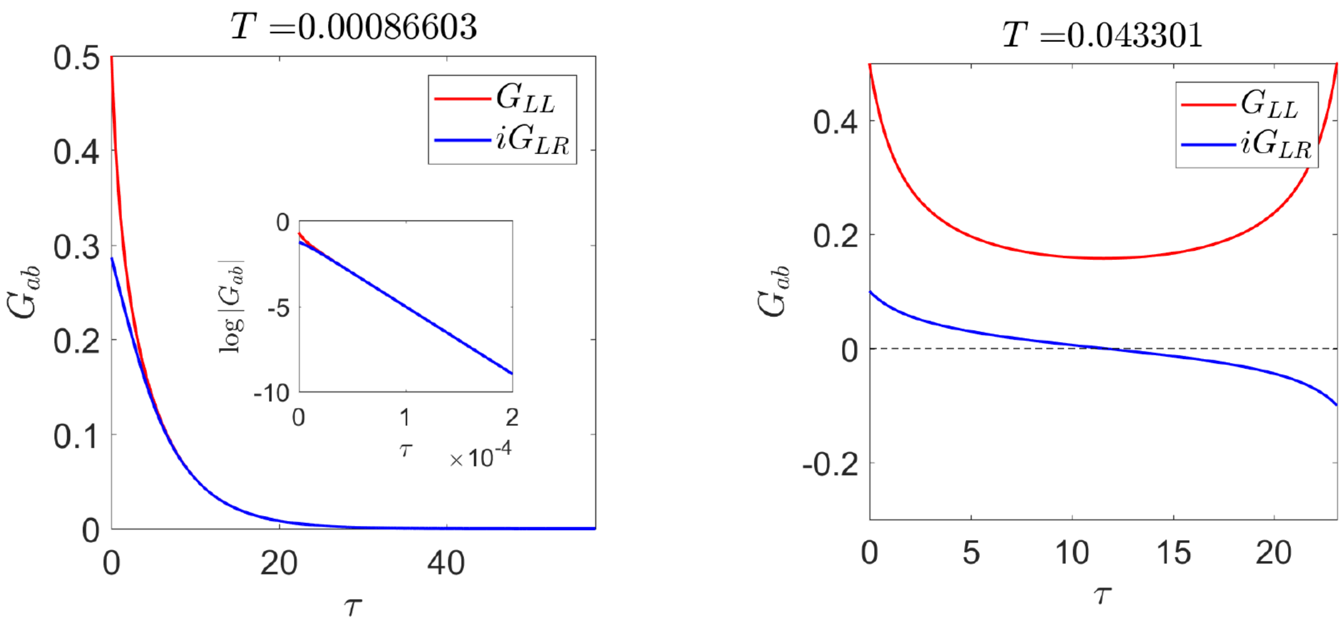


$$E_n = \sqrt{2-2\Delta}\left(n+\frac{1}{2}\right) \qquad \widetilde{E_n} = n + \widetilde{\Delta}$$



Beyond low energy: Schwinger-Dyson equation

- $H = \sum J_{i_1 i_2 \ldots i_q} \left(i^{\frac{q}{2}} \chi_{i_1 L} \chi_{i_2 L} \ldots \chi_{i_q L} + i^{-\frac{q}{2}} \chi_{i_1 R} \chi_{i_2 R} \ldots \chi_{i_q R} \right) + i\mu \sum_i \chi_{iL} \chi_{iR}$
- With the bilinear coupling, Schwinger-Dyson equation still apply
- $G(\omega_n) = (i\omega_n \mathbb{I} - \Sigma)^{-1}$
- $\Sigma_{\text{ab}}(\tau) = J^2 G_{ab}(\tau)^{q-1} + i\mu \epsilon_{ab} \delta(\tau)$
- Numerical solution in general μ, β, J

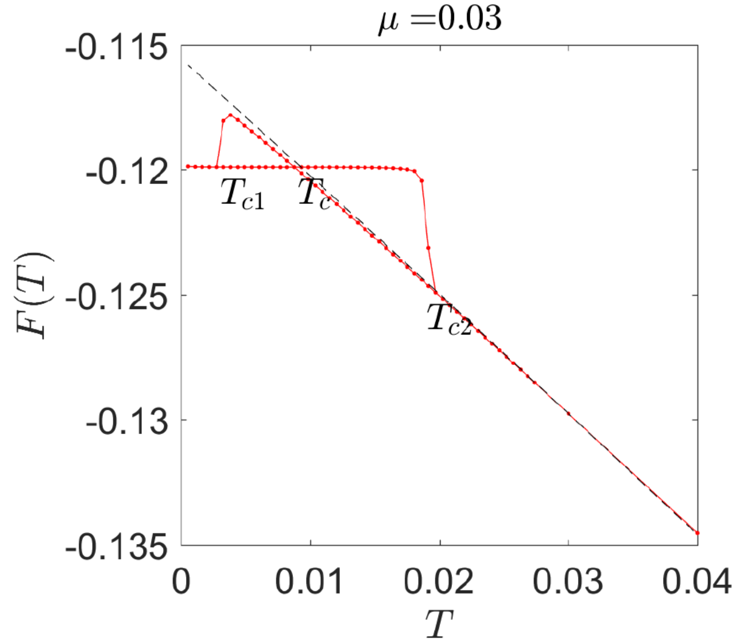


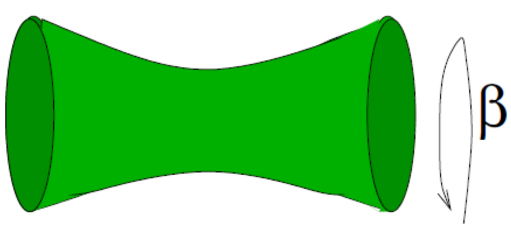
Large q limit

- Large q limit allows an analytic solution
- $G_{LL}(\tau_1,\tau_2) = \frac{1}{2}\operatorname{sgn}(\tau_{12})\, e^{\frac{g_{LL}(\tau_1,\tau_2)}{q}}, G_{LR}(\tau_1,\tau_2) = \frac{i}{2} e^{\frac{g_{LR}(\tau_1,\tau_2)}{q}}$
- $S_{eff}(g_{LL},g_{LR}) = \frac{N}{q^2}\Big[\int d\tau_1 d\tau_2 \big(\partial_1 g_{LL}\partial_2 g_{LL} - \partial_1 g_{LR}\partial_2 g_{LR} + J^2 e^{g_{LL}} + J^2 e^{g_{LR}}\big) + \hat{\mu}\int d\tau g_{LR}(\tau,\tau)\Big]$
- $\mu = \frac{\hat{\mu}}{q}$
- Zero temperature solution
- $e^{g_{LL}} = \frac{\alpha^2}{J^2 \sinh^2(\alpha|\tau_{12}|+\gamma)}, e^{g_{LR}} = \frac{\alpha^2}{J^2 \cosh^2(\alpha|\tau_{12}|+\gamma)}$
- $\alpha = \alpha(\mu), \gamma = \gamma(\mu)$
- For small μ, $\alpha \simeq \sqrt{\frac{\mu J}{2}}, \gamma \simeq \frac{\alpha}{J}$. Consistent with AdS2

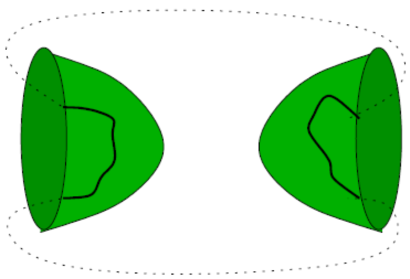
Hawking-Page-like transition

- Finite temperature of the coupled model
- First order phase transition for all q
- Low T phase: thermal gas in AdS2
- High T phase: μ term not important, two black holes, with coupled matter field





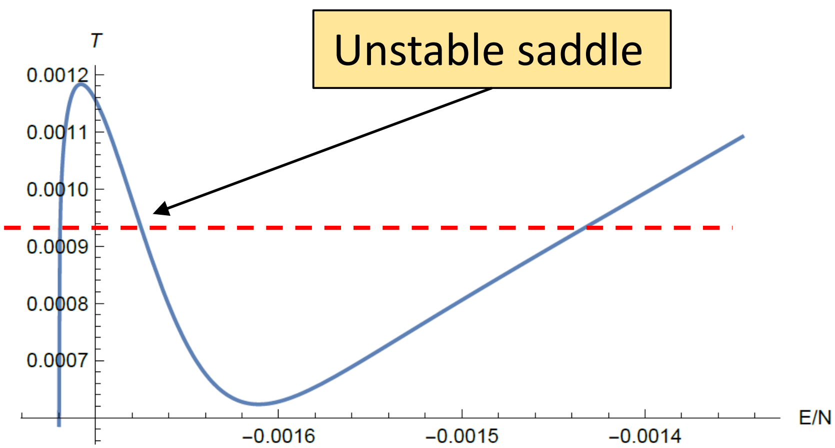
Low T

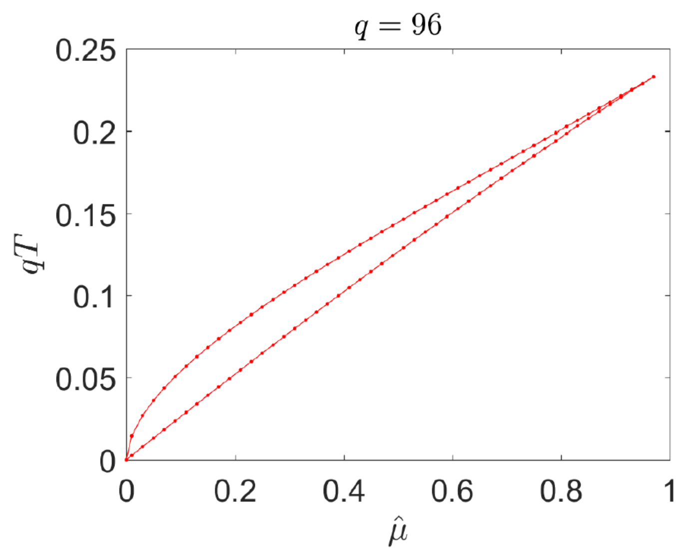


high T

Small black hole

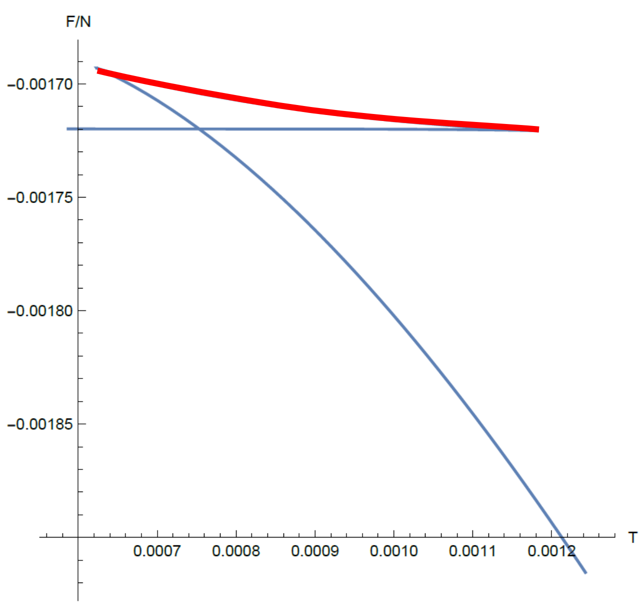
- If the transition is like Hawking-Page, is there a small black hole solution?
- Yes in large q
- $e^{g_{LL}} = \frac{\alpha^2}{J^2 \sinh^2(\alpha|\tau_{12}|+\gamma)}, e^{g_{LR}} = \frac{\tilde{\alpha}^2}{J^2 \cosh^2(\tilde{\alpha}|\tau_{12}|+\tilde{\gamma})}$ (+ periodic identification)
- For low temperature $\beta = \frac{q}{2\tilde{\alpha}(\sigma)} \log \frac{q}{\sigma}$
- Multiple solution at given β

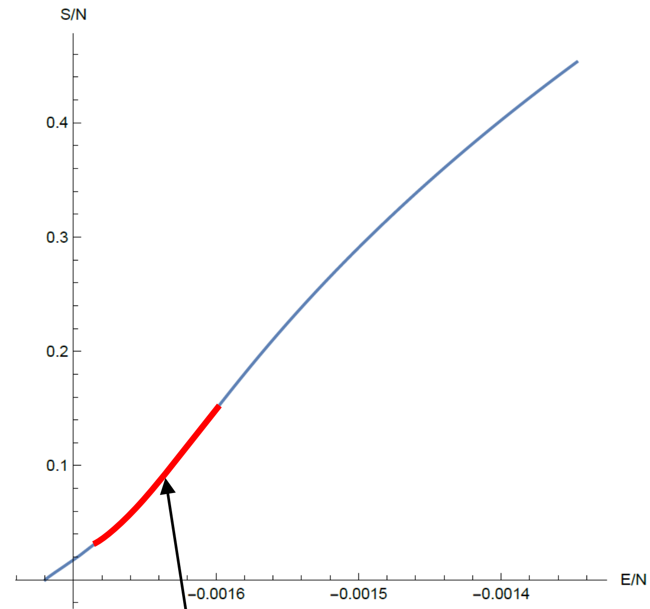




Small black hole

- The unstable saddle point is stable in microcanonical ensemble

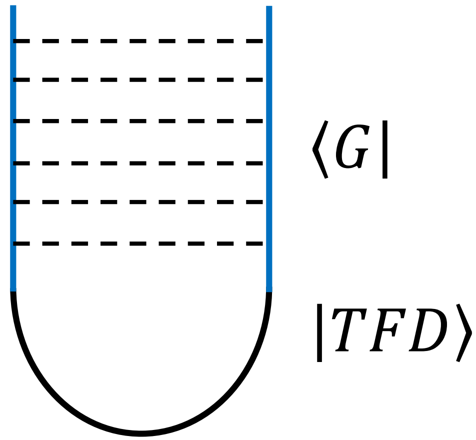


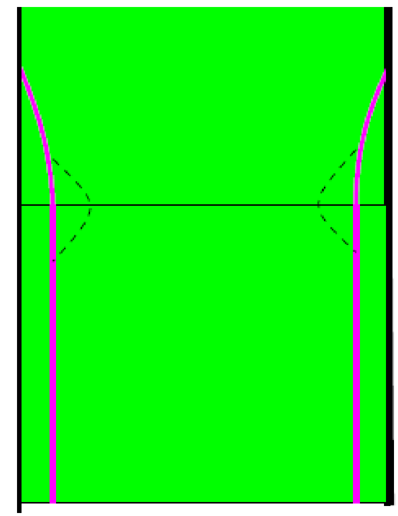


Negative specific heat region $\frac{\partial^2 S}{\partial E^2} = -\frac{\beta^2}{c_v} > 0$

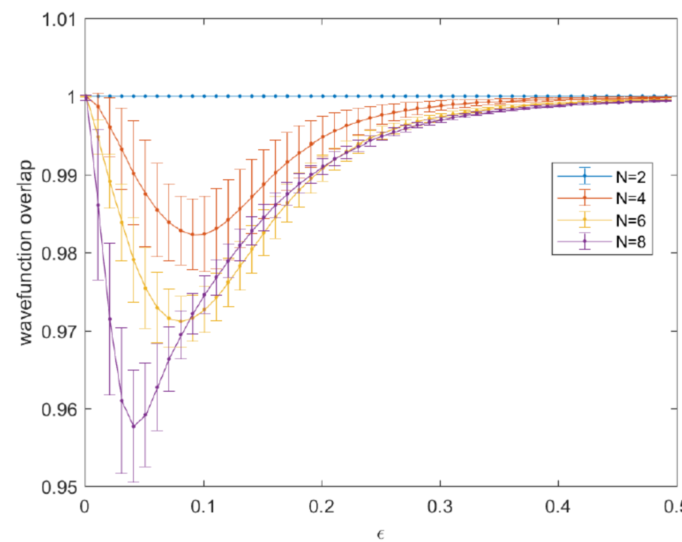
Overlap $\langle G|TFD\rangle$

- The overlap of ground state of coupled system and the thermal field double can be computed directly.
- Bulk picture: quench by switching off μ





- Low temperature: $|\langle G|TFD\rangle| \simeq e^{-\frac{N}{(\beta J)^2}\times const}$
- Large q: $|\langle G|TFD\rangle| \simeq e^{-\frac{N}{q^3}\times const}$
- Finite N numerics



Summary

- AdS2 JT gravity (+matter) with general boundary are dual to SYK model with relevant coupling.
- Eternal traversable worm hole in 2D
- Low energy spectrum fixed by approximate conformal symmetry
- Hawking-Page transition and small black hole phase
- Open questions:
  - Higher dimensional generalization
  - Evaporation of small black hole
  - Possible implication to the physics of black hole interior