Quantum closed superstrings & hyperbolic geometry

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String field theory

The scattering amplitudes in string theory are defined as the sum over all possible string diagrams, and string field theory decomposes this into a sum over Feynman diagrams.

Identifies certain string diagrams as elementary interactions, and the remaining string diagrams can be constructed by gluing elementary string diagrams with string propagators.

Using the elementary string diagrams, SFT constructs an action for string theory.

Plan

Discuss an explicit characterization of the elementary closed string diagrams, and describe the effective expression of the quantum action for closed superstring field theory, by exploring the hyperbolic geometry of string diagrams.

arXiv:1703.10563, 1708.04977, 1706.07366: Seyed Faroogh Moosavian, R.P.

To appear: R.P.

Elementary string diagrams

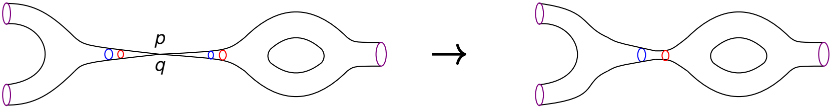
Elementary string diagrams can't be constructed by gluing other diagrams with propagators.

It is equipped with a choice of local coordinates w_i around the punctures and a distribution of picture changing operators (PCOs).

Local coordinates must be invariant under the permutation of the punctures, and the PCO distribution must satisfy the following requirements:

- Invariant under the permutation of the punctures (NS and R separately).
- Avoid the occurrence of unphysical poles.
- Invariant under the action of the large diffeomorphisms (MCG) on the string diagram.

Closed string propagator



A closed string propagator is a specific gluing of closed string diagrams.

Identifying the local coordinates w_p and w_q in the neighbourhoods of two punctures p and q on two different closed string diagrams using the following rule provide family of diagrams:

$$w_p w_q = t \qquad t = e^{-s+i\theta}$$

Gluing for $s \geq 0$ is the closed string propagator having length s.

String vertices

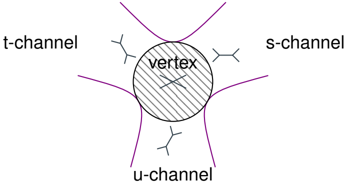
The set of all elementary string diagrams with h handles, m number of NS and n number of R punctures is called as the **string vertex** $\mathcal{V}_{h,m,n}$.

Closed SFT can be consistently quantized using the Batalin-Vilkovisky formalism only if $\mathcal{V}_{h,m,n}$ together with the diagrams F obtained by gluing elementary diagrams using string propagators generate a **single cover** of $\overline{\mathcal{M}}_{h,m+n}$:

$$\overline{\mathcal{M}}_{h,m+n} = \mathcal{V}_{h,m,n} \bigcup F^1_{h,m,n} \bigcup \cdots \bigcup F^Q_{h,m,n}$$

$F^J_{h,m,n}$: String diagram with J propagators. (Zwiebach)

Gluing compatibility



String vertices decomposes the moduli space into cells, and the integral of string measure over each cell is the contribution from a specific Feynman diagram.

This consistency condition demands that the local coordinates and PCO distribution on the diagrams at the boundary of a string vertex must match with that induced on the glued diagrams with zero length propagators.

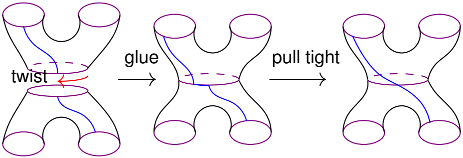
Minimal area metric

Complete set of consistent string vertices can be constructed by using string diagrams with metric of least possible area under the condition that lengths of all the nontrivial closed curves be longer than or equal to 2π. (Zwiebach)

Current understanding of minimal area metrics is limited, and have to depend on numerics. (N.Moeller; Wolf, Zwiebach; Headrick, Zwiebach)

Claim: complete set of string vertices can be constructed using string diagrams with metric having -1 constant curvature.

Hyperbolic geometry of string diagrams



A hyperbolic string diagram with n punctures and h handles, can be obtained by considering the proper discontinuous action of the Fuchsian group on the Poincaré upper half-plane.

Every such hyperbolic string diagrams can be obtained by the geometric sum of $2h - 2 + n$ number of hyperbolic pairs of pants.

Each attaching site has two parameters: the geodesic length ℓ of the boundary and the twist τ performed before gluing them. There are $3h - 3 + n$ attaching sites:

$$(\tau_j, \ell_j) \qquad 1 \leq j \leq 3h - 3 + n \qquad \tau_j \in \mathbb{R} \qquad \ell_j \in \mathbb{R}^+$$

Naive string vertices

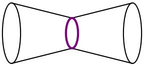
The unit area disc around a puncture on a hyperbolic diagram is isometric to a punctured disc with coordinates w and metric

$$\left(\frac{|dw|}{|w|\ln|w|}\right)^2$$

String vertex $\mathcal{V}^0_{h,m,n}$: the set of all inequivalent hyperbolic string diagrams having h handles, m- NS punctures and n- R punctures with no simple closed geodesic having length less than c_*. Each diagrams has local coordinates $e^{\frac{\pi^2}{c_*}} w$ around the punctures and a consistent distribution of $2h - 2 + m - \frac{n}{2}$ number of PCOs.

Does $\mathcal{V}^0_{h,m,n}$ together with the Feynman diagrams provide a single cover of $\overline{\mathcal{M}}_{h,m+n}$?

Gluing incompatibility of local coordinates



SFT gluing produces a string diagram having a collar with curvature accumulated on a curve.

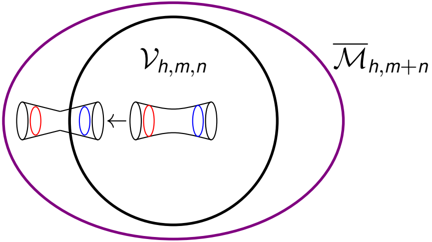
Metric around a puncture on a glued sting diagram with k zero length propagators:

$$\left(\frac{|dw|}{|w|\ln|w|}\right)^2 \left\{1 + \frac{c_*^2}{\ln|w|} Y + \mathcal{O}\left(c_*^3\right)\right\}$$

Y is the leading term $\frac{1}{3}\ln|w| \sum_{i=1}^{k} \left(E_{i,1} + E_{i,2}\right)$ around the puncture, where $E_{i,1}$, $E_{i,2}$ denote the Eisenstein series associated with the punctures that are being glued.

Local coordinates on the diagrams in $\mathcal{V}^0_{h,m,n}$ is not gluing compatible $\Rightarrow$ $\mathcal{V}^0_{h,m,n}$ together with the Feynman diagrams does not provide a single cover of $\overline{\mathcal{M}}_{h,m+n}$

Gluing compatible local coordinates



Violation of gluing compatibility is of the order $\mathcal{O}\left(c_*^2\right)$.

Assume that $c_* \ll 1 \Rightarrow$ make string vertex very large such that it cover most of the moduli space.

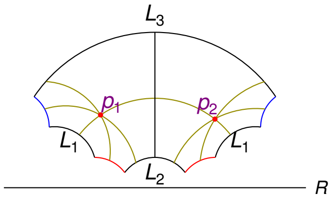
Modify the local coordinates continuously on the elementary string diagrams that belong to a thin neighbourhood of the boundary $\mathcal{V}^0_{h,m,n}$, such that at the boundary local coordinates matches to $\mathcal{O}\left(c_*^2\right)$ with that on the glued diagrams.

Consistent distribution of PCOs

The PCO distribution on an elementary string diagram:

- Must be invariant under the action of elements in MCG.
- Must be invariant under the permutation of the punctures (NS and R separately).
- Must be compatible with the gluing.

PCO distribution on a thrice-punctured sphere



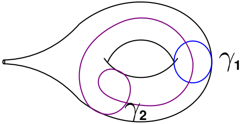
Need to insert a PCO on a thrice-punctured sphere if all the three punctures carry NS states.

Thrice-punctured sphere is a pair of pants with vanishing border lengths. PCO distribution on a pair of pants can be described by specifying the locations in it's fundamental domain: union of two hyperbolic hexagons in $\mathbb{H}$.

Distribution symmetric with respect boundaries, and interchange of right and left hexagons:

$$\frac{\mathcal{X}(p_1) + \mathcal{X}(p_2)}{2} \qquad p_1, p_2 : \quad \text{the centroids of right and left hexagons}$$

PCO distribution on a once-punctured torus



Fundamental domain of a once-punctured torus is obtained by first cutting it along a simple closed geodesic, then scissor the resulting once punctured-sphere with two borders.

Infinite number of fundamental domains by choosing different simple closed geodesics. All of them can be mapped to each other by the action of MCG.

Distributing PCO by choosing points in the fundamental domain also require specifying the simple closed geodesic along which we cut the surface, which is not an MCG invariant data.

PCO distribution obtained by choosing locations in a fundamental domain is not invariant under MCG.

MCG invariant PCO distribution

First distribute PCO in one fundamental domain and then average over all the fundamental domains obtained by cutting the surface along different simple closed geodesic that are MCG equivalent. The following distribution is manifestly MCG invariance:

$$\sum_{h \in \mathrm{MCG}} G(l_{h\cdot\gamma}, \tau_{h\cdot\gamma}) Q(c_*) \left\{ \frac{\chi(p_1^{h\cdot\gamma}) + \chi(p_2^{h\cdot\gamma})}{2} \right\}$$

where $\quad G(l_\gamma, \tau_\gamma) = \frac{2\,\mathrm{sinc}^2(\tau_\gamma)}{1+e^{l_\gamma}} \qquad Q(c_*) = 1 + \frac{1}{2}c_* + \frac{1}{4}c_*^2$

Infinite sums often diverge, but McShane Identity for a simple closed geodesic γ on a once-punctured torus makes the distribution well defined:

$$\sum_{h \in \mathrm{MCG}} \frac{2\,\mathrm{sinc}^2\left(\tau_{h\cdot\gamma}\right)}{1 + e^{l_{h\cdot\gamma}}} = 1$$

Gluing compatible PCO distribution

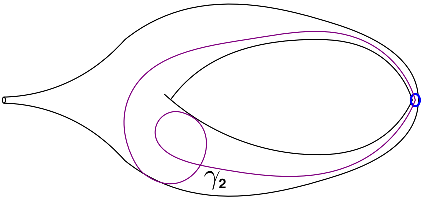


Figure: $l_{\gamma_2} \longrightarrow e^{1/c_*}$ when $l_{\gamma_1} \longrightarrow c_*$

When $l_\gamma \longrightarrow c_*$ the PCO distribution becomes that on the once-punctured torus with zero length NS propagator:

$$\sum_{h \in \mathrm{MCG}} G(l_{h\cdot\gamma}, \tau_{h\cdot\gamma}) Q(c_*) \left\{ \frac{\chi(p_1^{h\cdot\gamma}) + \chi(p_2^{h\cdot\gamma})}{2} \right\} \rightarrow \frac{\chi(p_1^{\gamma}) + \chi(p_2^{\gamma})}{2} + \mathcal{O}\left(c_*^3\right)$$

Improved string vertices

It is possible to systematically construct manifestly MCG invariant PCO distributions $\mathbf{K}$ on any hyperbolic string diagrams which is gluing compatible up to order $\mathcal{O}\left(c_*^2\right)$, using the generalization of McShane identity:

$$\mathbf{K} = \sum_{\mathbf{TG}} \sum_{h \in \mathrm{MCG}} \mathbf{X_{TG}}$$

First sum is over the set of all trivalent graphs $\mathbf{TG}$ with h handles and $m + n$ external legs.

$\mathcal{V}^2_{h,m,n}$ **provide a consistent closed superstring field theory for infintesimal** c_* **.**

Making c_* very small a valid operation, since none of the physical quantities in string field theory depends on it.

Effective expression for quantum BV action

$$S = \text{Kinetic term} + \sum_{(m+n,\,h)=(1,\,0)}^{(\infty,\infty)} \frac{g_s^{2h-2+m+n}}{m+n!}\{\Psi^{m+n}\}_h$$

$\{\cdots\}_h$ is obtained by integrating the h loop off-shell superstring measure $\Omega(\cdots)$ over the region inside the moduli space $\mathcal{MV}^2_{h,m,n}$ covered by the string vertex $\mathcal{V}^2_{h,m,n}$:

$$\begin{aligned}\{\cdots\}_h &= \int_{\mathcal{MV}^2_{h,m,n}} \Omega(\cdots)\\ &= \sum_{\mathbf{TG}} \sum_{h\in \mathrm{MCG}} \int_{\mathcal{MV}^2_{h,m,n}} \Omega^{\mathbf{TG}}(\cdots)\\ &= \sum_{\mathbf{TG}} \int_{\ell_1=c_*}^{\infty} \int_{\tau_1=-\infty}^{\infty} \cdots \int_{\ell_Q=c_*}^{\infty} \int_{\tau_Q=-\infty}^{\infty} \Omega^{\mathbf{TG}}(\cdots)\end{aligned}$$

String perturbation theory

The on-shell string amplitude is obtained by integrating the h loop on-shell superstring measure $\Lambda(\cdots)$ over the region inside the moduli space $\overline{\mathcal{M}}_{h,m+n}$:

$$\int_{\mathcal{M}_{h,m+n}} \Lambda(\cdots) = \sum_{\mathbf{TG}} \int_{\ell_1=0}^{\infty} \int_{\tau_1=-\infty}^{\infty} \cdots \int_{\ell_Q=0}^{\infty} \int_{\tau_Q=-\infty}^{\infty} \Lambda^{\mathbf{TG}}(\cdots)$$

This form is very convenient for analyzing the field theory limit of superstring amplitudes.

Summary

Hyperbolic geometry can be used to construct a calculable closed superstring field theory.

Provide efficient tools for computing the amplitudes using superstring perturbation theory.