N=1 Lagrangians for N=2 “non-Lagrangian” theories

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Based on work with

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1606.05632, 1607.04281, 1610.05311, 1707.04751, 1806.08353

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What is the “simplest” interacting 4d N=2 SCFT?

A) Argyres-Douglas theory (very likely)

Argyres-Douglas theory

- Theory describing a special point in the Coulomb branch of N=2 SU(3) SYM or N=2 SU(2) SQCD.
  [Argyres-Douglas 95] [Argyres-Plesser-Seiberg-Witten 95]
- At this special point, mutually non-local electromagnetically charged particles become massless.
- It is a strongly-coupled N=2 SCFT with no tunable coupling. Commonly referred to as a "**non-Lagrangian theory**"
- Many generalizations. [Cecotti-Neitzke-Vafa 10][Cecotti-del Zotto 12][Xie 12]…
- Its Coulomb phase is well-understood, but the conformal phase is less-understood. [Cornagliotto, Lemos, Liendo 17][Talk by Lemos]

Properties of the H_0 Argyres-Douglas theory

- There is a chiral operator of dimension 6/5 parametrizing the Coulomb branch.
- Central charges: $$a = \frac{43}{120}\,, \quad c = \frac{11}{30}$$ [Aharony-Tachikawa 07] [Shapere-Tachikawa 08]
- The central charge c above is the minimal value of any interacting N=2 SCFT! [Liendo-Ramirez-Seo 15] $$c \geq \frac{11}{30}$$
- The 2d chiral algebra corresponding to the AD theory is given by a non-unitary Virasoro minimal model.

[Beem-Lemos-Liendo-Rastelli-van Rees 13]
[Cordova-Shao 15]

Is it possible to write a Lagrangian for the ‘simplest 4d N=2 SCFT’?

Challenges:

- Mutually non-local particles.
- Chiral operators of fractional dimensions.

A) Sacrifice manifest (super)symmetry

N=1 gauge theory flowing to the $H_0=(A_1,A_2)$ SCFT

[Maruyoshi-JS 16]

Matter content

	q	q'		M	X
SU(2)	2	2	adj	1	1

Superpotential

$$W = \phi q q + M\phi q' q' + X\phi^2$$

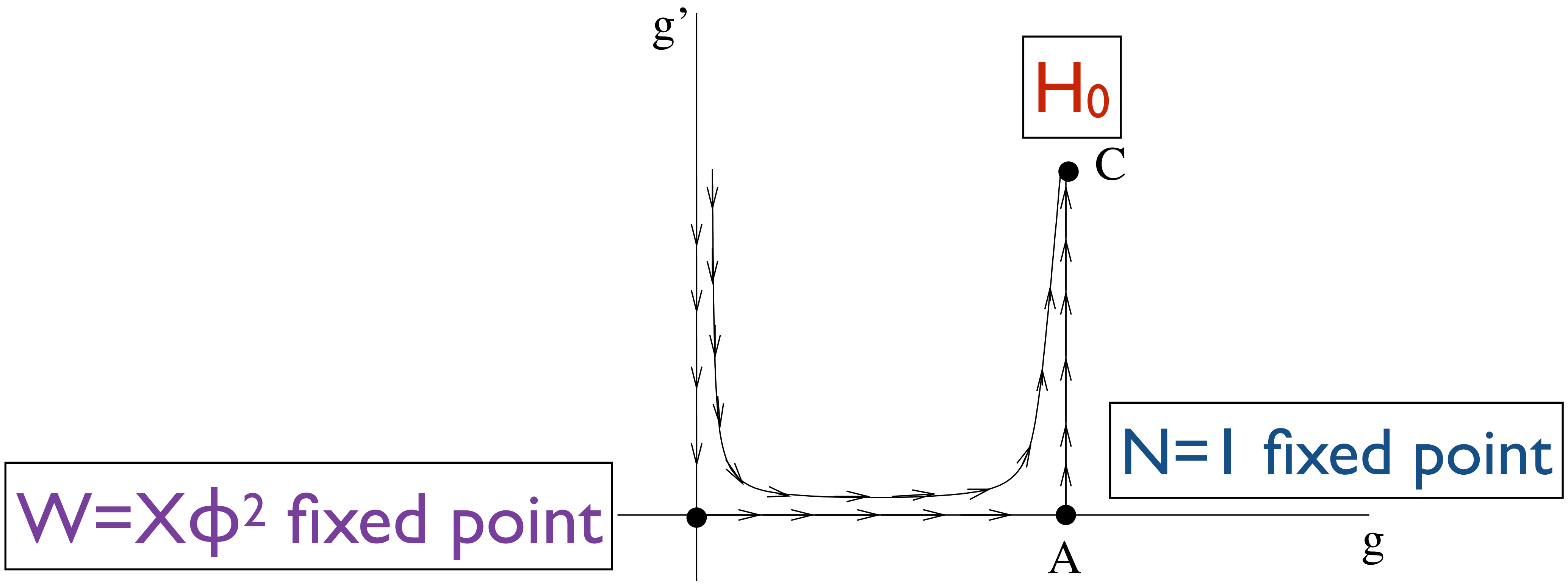
This theory has an anomaly free U(1) global symmetry that can be mixed with R-symmetry. R-charges fixed by a-maximization.

[Intriligator-Wecht 03]

RG Flow to the H_0 theory

[Maruyoshi-JS][Maruyoshi-Nardoni-JS]

$$W = g\phi qq + g'M\phi q'q' + \lambda X\phi^2$$



@C, we get: $a = \frac{43}{120}\ , \quad c = \frac{11}{30} \quad \Delta(M) = \frac{6}{5}$

Agrees with that of the Argyres-Douglas theory!

N=1 gauge theory flowing to the $H_1=(A_1,A_3)$ theory

- Matter contents:

	q	q′		M	X
SU(2)	2	2	adj	1	1

- Interaction: $$W = Mqq' + X\phi^2$$

This theory has SU(2)xU(1) flavor symmetry.
The U(1) symmetry can be mixed with R-symmetry.

@IR, we get: $a = \frac{11}{24}, \quad c = \frac{1}{2}, \quad \Delta(M) = \frac{4}{3}$

Checks & applications of the N=1 Lagrangian for the AD theory

- Moduli space, chiral ring agree with the known results.
- The full superconformal index of the AD theory. Can be compared against the Schur/Macdonald limits of the index computed in [Cordova-Shao][Buican-Nishinaka][JS]
- 3d Argyres-Douglas theory: 3d N=2→N=4 [Benvenuti-Giacomelli]
- More SUSY partition functions for the AD theory [Fredrickson-Pei-Yan-Ye][Gukov][Fluder-JS]

Where are these 'Lagrangians' coming from?
Is there any organizing principle?

N=1 Deformations of N=2 SCFT with global symmetry

- Consider N=2 SCFT $\mathcal{T}_{UV}$ with non-abelian global symmetry $\mathcal{F}$.
- It has a moment map operator μ valued in the adjoint of $\mathcal{F}$.
- Add a chiral multiplet M in the adjoint of $\mathcal{F}$ and the following superpotential:

$$W = \mathrm{Tr}(M\mu)$$

- SU(2)xU(1) R-symmetry broken to $U(1)_R \times U(1)_A$

N=1 Deformation via Nilpotent Higgsing

[Gadde-Maruyoshi-Tachikawa-Yan]
[Agarwal-Bah-Maruyoshi-JS]
[Agarwal-Intriligator-JS]

- Now, we give a ***nilpotent vev*** to M.

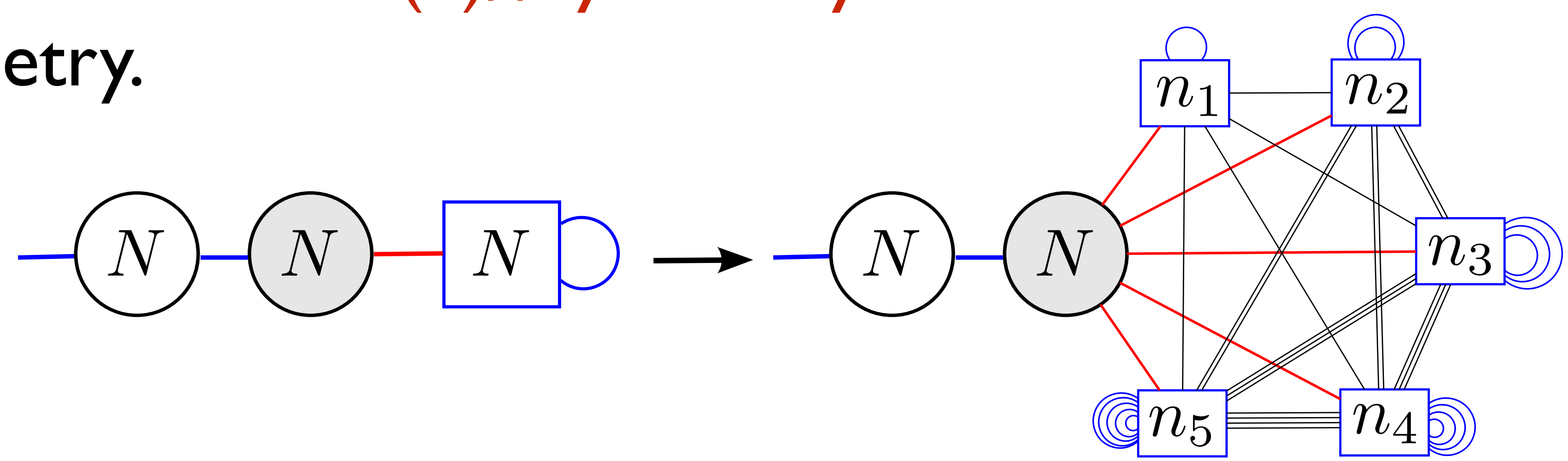
- The deformation triggers a flow to a new N=1 SCFT.

$$\mathcal{T}_{UV} \rightsquigarrow \mathcal{T}_{IR}[\mathcal{T}_{UV}, \rho]$$

- Nilpotent elements are classified by the SU(2) embeddings ρ: SU(2) → $\mathcal{F}$.

$$\langle M \rangle = \rho(\sigma^+)$$

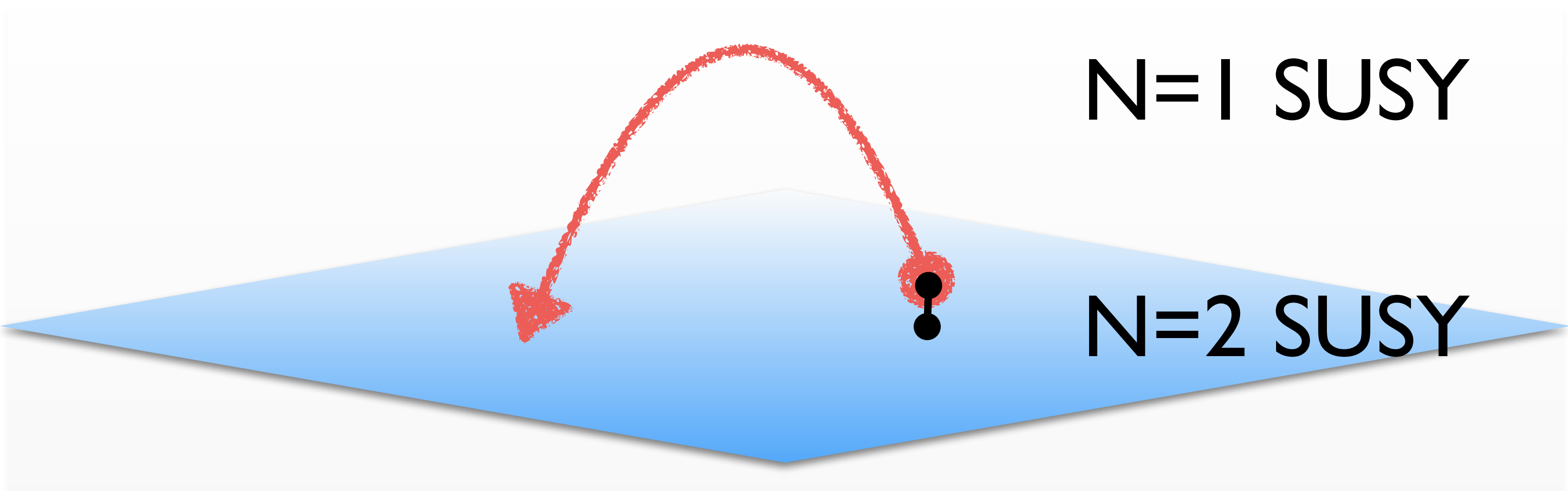
- It preserves the $U(1)_A$ symmetry that can be mixed with R-symmetry.



Results

[Maruyoshi-JS][Agarwal-Maruyoshi-JS]

- For a number of cases, **Supersymmetry enhances to N=2** at the fixed point.
  - N=1 RG flows between (known) N=2 SCFTs
  - N=1 deformed Lagrangian N=2 SQCD flows to the "non-Lagrangian" Argyres-Douglas (AD) theory!



Deforming SU(N) N_f=2N: $\mathcal{F}$=SU(2N)

$SU(2N)$	$\rho : SU(2) \hookrightarrow SU(2N)$	a	c	4d $\mathcal{N}=2$ SUSY
$SU(4)$	$[1^4]$	$\frac{23}{24}$	$\frac{7}{6}$	Yes; $N_c=2,\ N_f=4$
	$[3,1]$	$\frac{7}{12}$	$\frac{2}{3}$	Yes; (A_1, D_4) AD th.
	$[4]$	$\frac{11}{24}$	$\frac{1}{2}$	Yes; (A_1, A_3) AD th.
$SU(6)$	$[1^6]$	$\frac{29}{12}$	$\frac{17}{6}$	Yes; $N_c=3,\ N_f=6$
	$[5,1]$	$\frac{13}{12}$	$\frac{7}{6}$	Yes; (A_1, D_6) AD th.
	$[6]$	$\frac{11}{12}$	$\frac{23}{24}$	Yes; (A_1, A_5) AD th.
$SU(8)$	$[1^8]$	$\frac{107}{24}$	$\frac{31}{6}$	Yes; $N_c=4,\ N_f=8$
	$[2,1^6]$	$\frac{73801}{17424}$	$\frac{43121}{8712}$	?
	$[4,4]$	$\frac{9097}{3888}$	$\frac{5129}{1944}$	?
	$[7,1]$	$\frac{19}{12}$	$\frac{5}{3}$	Yes; (A_1, D_8) AD th.
	$[8]$	$\frac{167}{120}$	$\frac{43}{30}$	Yes; (A_1, A_7) AD th.
$SU(10)$	$[1^{10}]$	$\frac{247}{24}$	$\frac{71}{6}$	Yes; $N_c=5,\ N_f=10$
	$[5,1^5]$	$\frac{5553943}{1383123}$	$\frac{6257387}{1383123}$	?
	$[5,3,1^2]$	$\frac{92540867}{24401712}$	$\frac{52091009}{12200856}$	?
	$[9,1]$	$\frac{25}{12}$	$\frac{13}{6}$	Yes; (A_1, D_{10}) AD th.
	$[10]$	$\frac{15}{8}$	$\frac{23}{12}$	Yes; (A_1, A_9) AD th.
$SU(12)$	$[1^{12}]$	$\frac{247}{24}$	$\frac{71}{6}$	Yes; $N_c=6,\ N_f=12$
	$[4^3]$	$\frac{754501}{138384}$	$\frac{424727}{69192}$	?
	$[11,1]$	$\frac{31}{12}$	$\frac{8}{3}$	Yes; (A_1, D_{12}) AD th.
	$[12]$	$\frac{397}{168}$	$\frac{101}{42}$	Yes; (A_1, A_{11}) AD th.

Here we list some of the deformations that gives rational central charges.

Those with “?" have N=1 SUSY. [Evtikhiev]

Other deformations give irrational central charges, therefore they flow to N=1 theories.

Deforming Sp(N),N_f=2N+2: $\mathcal{F}$=SO(4N+4)

$SO(4N+4)$	$\rho : SU(2) \hookrightarrow SO(4N+4)$	a	c	4d $\mathcal{N}=2$ SUSY
$SO(8)$	$[1^8]$	$\frac{23}{24}$	$\frac{7}{6}$	Yes; $N_c = 1,\ N_f = 8$
	$[3^2, 1^2]$	$\frac{7}{12}$	$\frac{2}{3}$	Yes; (A_1, D_4) AD th.
	$[4,4] \equiv [5,1^3]$	$\frac{11}{24}$	$\frac{1}{2}$	Yes; (A_1, D_3) AD th.
	$[5,3]$	$\frac{6349}{13872}$	$\frac{3523}{6936}$	?
	$[7,1]$	$\frac{43}{120}$	$\frac{11}{30}$	Yes; (A_1, A_2) AD th.
$SO(12)$	$[1^{12}]$	$\frac{37}{12}$	$\frac{11}{3}$	Yes; $N_c = 2,\ N_f = 12$
	$[4^2, 2^2]$	$\frac{105027}{59536}$	$\frac{61145}{29768}$	?
	$[9, 1^3]$	$\frac{19}{20}$	1	Yes; (A_1, D_5) AD th.
	$[11, 1]$	$\frac{67}{84}$	$\frac{17}{21}$	Yes; (A_1, A_4) AD th.
$SO(16)$	$[1^{16}]$	$\frac{51}{8}$	$\frac{15}{2}$	Yes; $N_c = 3,\ N_f = 16$
	$[5, 1^{11}]$	$\frac{109031}{27744}$	$\frac{123889}{27744}$	?
	$[5, 3^3, 1^2]$	$\frac{18250741}{5195568}$	$\frac{10440877}{2597784}$	?
	$[13, 1^3]$	$\frac{81}{56}$	$\frac{3}{2}$	Yes; (A_1, D_7) AD th.
	$[15, 1]$	$\frac{91}{72}$	$\frac{23}{18}$	Yes; (A_1, A_6) AD th.
$SO(20)$	$[1^{20}]$	$\frac{65}{6}$	$\frac{38}{3}$	Yes; $N_c = 4,\ N_f = 20$
	$[2^2, 1^{16}]$	$\frac{4181}{400}$	$\frac{2463}{200}$	?
	$[3^4, 2^4]$	$\frac{29}{4}$	$\frac{133}{16}$	?

Deforming SO(N) N_f=N-2: $\mathcal{F}$=Sp(N-2)

$Sp(N-2)$	$\rho : SU(2) \hookrightarrow Sp(N-2)$	a	c	4d $\mathcal{N}=2$ SUSY
$Sp(2)$	$[1^4]$	$\frac{19}{12}$	$\frac{5}{3}$	Yes; $N_c=4,\ N_f=4$
	$[2,1^2]$	$\frac{10111}{7056}$	$\frac{5381}{3528}$	?
$Sp(3)$	$[1^6]$	$\frac{65}{24}$	$\frac{35}{12}$	Yes; $N_c=5,\ N_f=6$
	$[4,1^2]$	$\frac{325}{192}$	$\frac{341}{192}$	?
$Sp(4)$	$[1^8]$	$\frac{33}{8}$	$\frac{9}{2}$	Yes; $N_c=6,\ N_f=8$
$Sp(5)$	$[1^{10}]$	$\frac{35}{6}$	$\frac{77}{12}$	Yes; $N_c=7,\ N_f=10$
$Sp(6)$	$[1^{12}]$	$\frac{47}{6}$	$\frac{26}{3}$	Yes; $N_c=8,\ N_f=12$
	$[2^2,1^8]$	$\frac{589093}{80688}$	$\frac{329335}{40344}$	?
	$[4,1^8]$	$\frac{13065}{2312}$	$\frac{7085}{1156}$	?
$Sp(7)$	$[1^{14}]$	$\frac{81}{8}$	$\frac{45}{4}$	Yes; $N_c=9,\ N_f=14$
	$[5^2,1^4]$	$\frac{59094550}{10978707}$	$\frac{129141025}{21957414}$	?
	$[6,3^2,2]$	$\frac{375975613}{72745944}$	$\frac{406255085}{72745944}$	?
$Sp(8)$	$[1^{16}]$	$\frac{305}{24}$	$\frac{85}{6}$	Yes; $N_c=10,\ N_f=16$
	$[4^2,2^2,1^4]$	$\frac{389}{48}$	$\frac{53}{6}$	?
	$[5^2,3^2]$	$\frac{30593927}{4642608}$	$\frac{16735805}{2321304}$	?
	$[5^2,4,1^2]$	$\frac{28118905}{4348848}$	$\frac{3828919}{543606}$	?

No non-trivial N=2 fixed point!

Is there any pattern in the SUSY enhancement?

Chiral Algebra associated to $\mathcal{T}_{\mathrm{UV}}$

[Beem-Lemos-Liendo-Peelaers-Rastelli-van Rees]

$$\mathcal{T}_{UV} \mapsto \chi_{2d}[\mathcal{T}_{UV}]$$

- For any 4d N=2 SCFT, there is a subsector described by a chiral algebra with

$$c_{2d} = -12 c_{4d}, \qquad k_{2d} = -\frac{1}{2} k_{4d} \ .$$

- If the chiral algebra is given by the affine Kac-Moody algebra $\hat{\mathscr{F}}_{k_{2d}}$, the stress tensor is given by the Sugawara tensor with the central charge

$$c_{\mathrm{Sugawara}} = \frac{k_{2d} \dim \mathscr{F}}{k_{2d} + h^{\vee}}$$

When does SUSY enhances?

[Agarwal-Maruyoshi-JS][Agarwal-Sciarappa-JS]
[Giacomelli]

$$\mathcal{T}_{UV} \rightsquigarrow \mathcal{T}_{IR}[\mathcal{T}_{UV}, \rho]$$

By studying a large set of Lagrangian/non-Lagrangian theories, we observe that the SUSY is enhanced in the IR if and only if

- $\mathcal{T}_{\mathrm{UV}}$ satisfies $c_{2d}[\mathcal{T}_{\mathrm{UV}}] = c_{\mathrm{Sugawara}(+n)}$ & $\mathcal{F}$ is of ADE type (can have U(1)'s)
- Either of the two cases:

1. ρ is the principal embedding.
2. ρ is the 'next to principal' and $\mathcal{T}_{\mathrm{UV}}$ saturates the flavor central charge bound. $k_F \geqq k_{F,b}$ [BLLPRvR][Lemos-Liendo]

WHY?

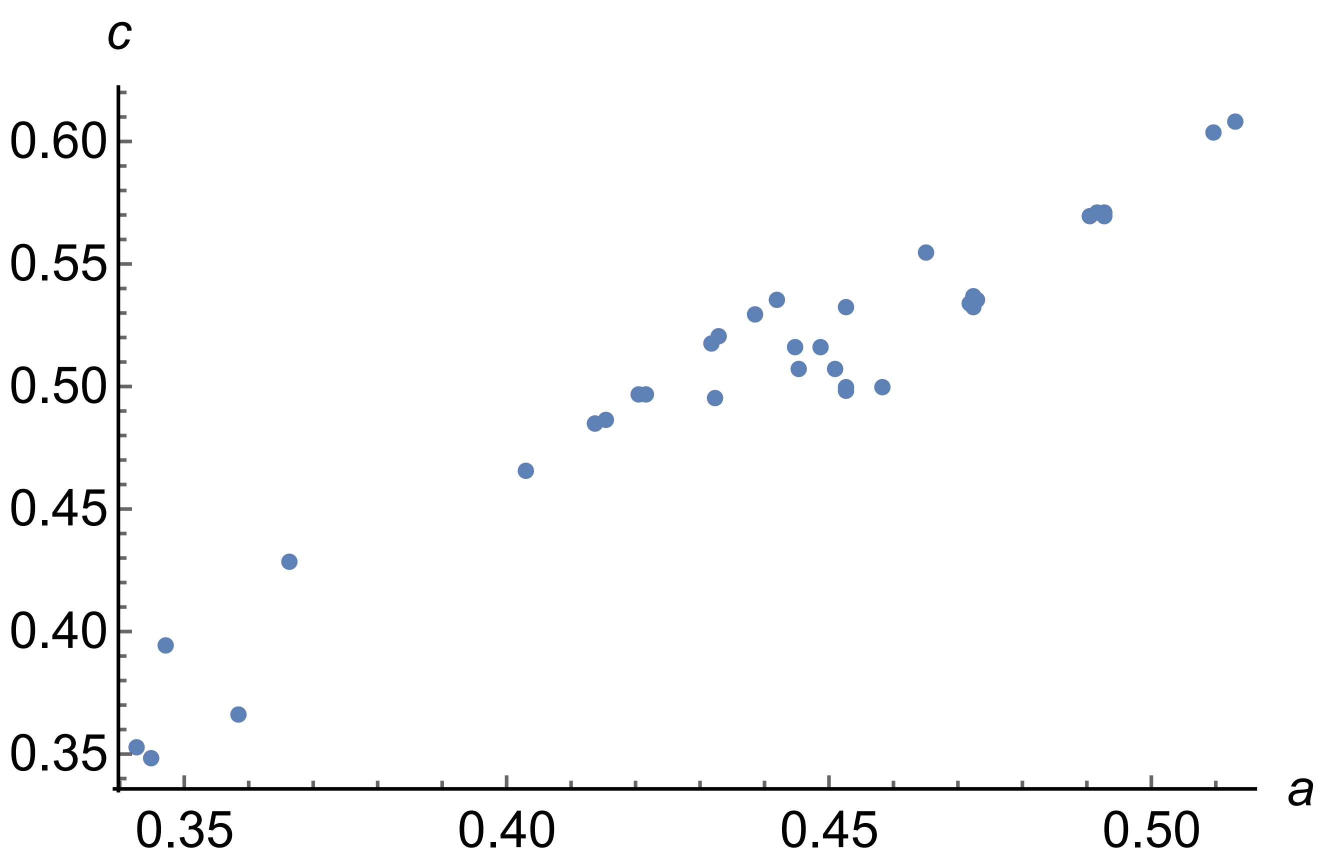
What about the minimal 4d $N=1$ SCFT?

Minimal 4d N=1 SCFT?

- There is no analytic bound on the value of a or c.
- There is a candidate minimal SCFT suggested by conformal bootstrap with c~0.11. But no explicit construction of such theory. [Poland-Stergiou][Li-Meltzer-Stergiou]
- We explored a large set of SCFTs with Lagrangian descriptions by considering a simple setup.
- SU(2) adjoint SQCD with $N_f=1$ + gauge singlets with all possible superpotential couplings

A Landscape of Simple SCFTs

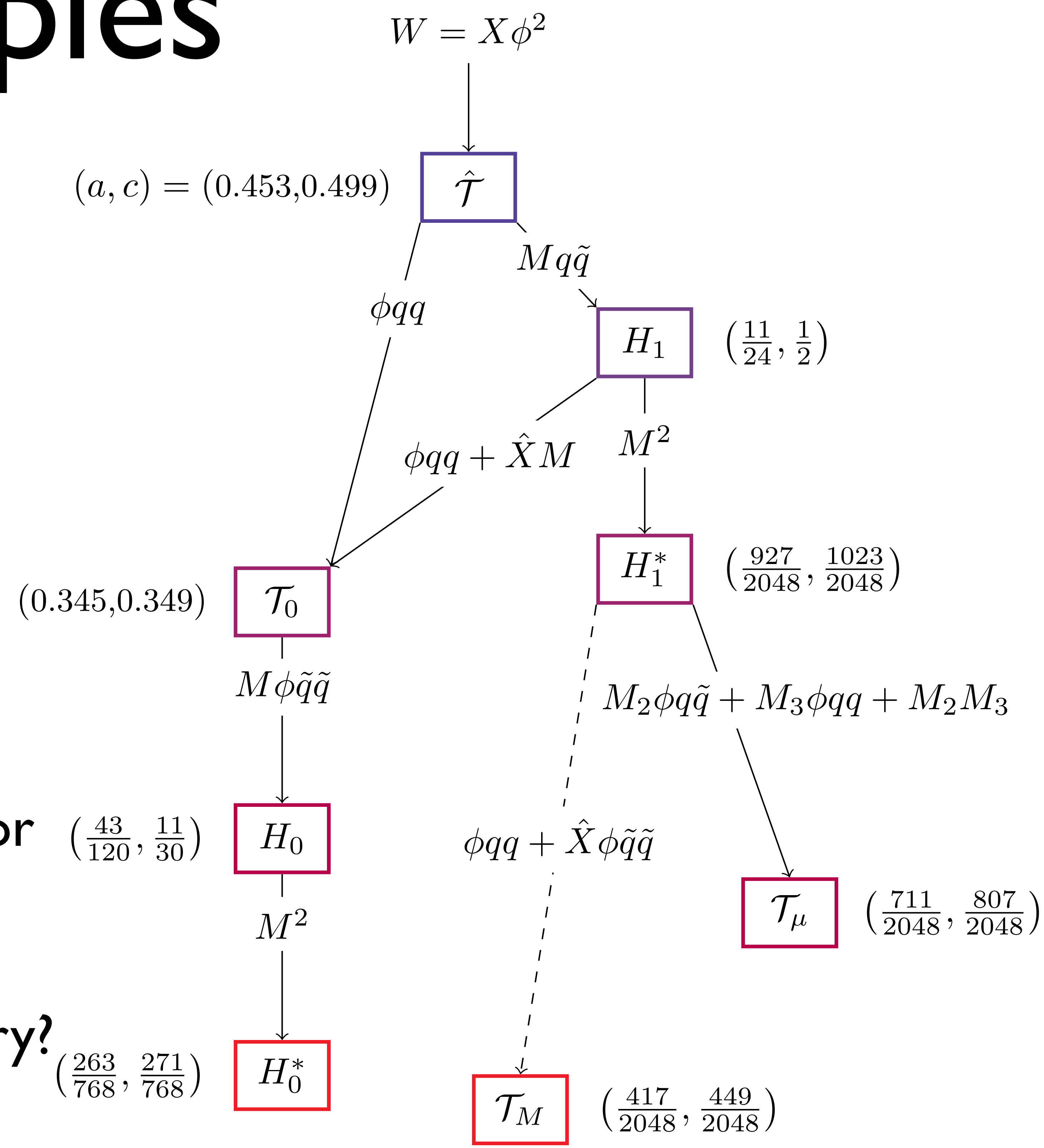
[Maruyoshi-Nardoni-JS]



- We found 35 fixed points having small central charges.
- They pass a number of consistency checks: central charge bounds, unitarity constraints, index
- a/c lie in a narrow range with mean value and std
  $$0.8733 \pm 0.0398$$

Some examples

- H_0^* has the smallest value of *a*.
  [Xie-Yonekura][Buican-Nishinaka]
- $\mathcal{T}_0$ and H_1^* have the smallest a with U(1) and SU(2) flavor.
- $\mathcal{T}_0$ has the smallest value of *c*.
  [Benvenuti]
- $\mathcal{T}_M$ contains 'unphysical' operator that is not in the chiral ring.
  Is it really 'bad' or just 'ugly'?
  Can it be a new "minimal" theory?



Summary

Summary

- To a given N=2 SCFT $\mathcal{T}_{\text{UV}}$ with non-abelian global symmetry $\mathcal{F}$, one can obtain N=1 SCFT $\mathcal{T}_{\text{IR}}[\mathcal{T}_{\text{UV}},\rho]$ labelled by SU(2) embedding ρ of $\mathcal{F}$.

$$\mathcal{T}_{UV} \rightsquigarrow \mathcal{T}_{IR}[\mathcal{T}_{UV}, \rho]$$

- For some special cases, $\mathcal{T}_{\text{IR}}$ have enhanced N=2 SUSY.

- N=1 Lagrangian theories flowing to the N=2 Argyres-Douglas theories can be realized in this way.

- Many new "simple N=1 SCFTs" with small central charges can be constructed from a simple gauge theory setup.

Outlook

- When and why SUSY enhancement happens?
- Other 'non-Lagrangian' theories? general ADs, T_N, $\mathcal{N}$=3 cf) E_6, E_7, $R_{0,N}$ SCFT [Gadde-Razamat-Willett][Agarwal-Maruyoshi-JS]
- SUSY enhancements in other d? [Gaiotto-Komargodski-Wu] [Benini-Benvenuti][Gang-Yamazaki]
- What is the minimal N=1 SCFT?

Thank you!