M-theory S-Matrix from 3d SCFT

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Based on:

- arXiv:1711.07343 with N. Agmon and S. Chester
- arXiv:1804.00949 with S. Chester and X. Yin

Also:

- arXiv:1406.4814, arXiv:1412.0334 with S. Chester, J. Lee, and R. Yacoby
- arXiv:1610.00740 with M. Dedushenko and R. Yacoby

OIST, June 26, 2018

Motivation

- Learn about gravity / string theory / M-theory from CFT.
- 3d maximally supersymmetric ($\mathcal{N} = 8$) CFTs w/ gravity duals: explicit Lagrangians; no marginal coupling; SUSY.
- Most well-understood example:
  M-theory on $AdS_4 \times S^7 \Longleftrightarrow U(N)_k \times U(N)_{-k}$ ABJM theory at CS level $k = 1$.
- Last 10 years: progress in QFT calculations
  - using supersymmetric localization;
  - using conformal bootstrap in CFTs.
- **What do these calculations tell us about M-theory?**
- Example: S^3 partition function of ABJM theory can be written as an N-dim'l integral. What info about M-theory does it contain?

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M-theory S-matrix

- **This talk:** Reconstruct M-theory S-matrix perturbatively at small momentum (scatter gravitons and superpartners).
- Equivalently, reconstruct the derivative expansion of the M-theory effective action. Schematically,

$$S = \int d^{11}x \sqrt{g} \left[R + \text{Riem}^4 + \cdots + (\text{SUSic completion}) \right] .$$

- Restrict momenta to be in 4 out of the 11 dimensions.

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- From the 4d point of view, we can scatter:
  - graviton (1);
  - gravitinos (8);
  - gravi-photons (28);
  - gravi-photinos (56);
  - scalars ($70 = 35 + 35$)

- At **leading** order in small momentum (i.e. momentum squared), scattering amplitudes are those in $\mathcal{N} = 8$ SUGRA at tree level. Amplitude depends on the type of particle, e.g.

$$\mathcal{A}_{\text{SUGRA, tree}}(h^- h^- h^+ h^+) = \frac{\langle 12\rangle^4 [34]^4}{stu},$$

$$\mathcal{A}_{\text{SUGRA, tree}}(S_1 S_1 S_2 S_2) = \frac{tu}{s},$$

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Momentum expansion

- Momentum expansion takes a universal form (independent of the type of particle):

$$\mathcal{A} = \mathcal{A}_{\text{SUGRA, tree}} \Big(1 + \ell_p^6 f_{R^4}(s,t) + \ell_p^9 f_{\text{1-loop}}(s,t) + \ell_p^{12} f_{D^6 R^4}(s,t) + \ell_p^{14} f_{D^8 R^4}(s,t) + \cdots \Big) .$$

- $f_{D^{2n}R^4} =$ symmetric polyn in s, t, u of degree $n+3$
- Known from type II string theory + SUSY [Green, Tseytlin, Gutperle, Vanhove, Russo, Pioline, ...] :

$$f_{R^4}(s,t) = \frac{stu}{3 \cdot 2^7} , \qquad f_{D^6 R^4}(s,t) = \frac{(stu)^2}{15 \cdot 2^{15}} .$$

- $\ell_p^{10} f_{D^4 R^4}$ allowed by SUSY, but known to vanish.
- **This talk:** Reproduce f_{R^4} from 3d SCFT.

Flat space limit of CFT correlators

- Idea: Flat space scattering amplitudes can be obtained as limit of CFT correlators [Polchinski '99; Susskind '99; Giddings '99; Penedones '10; Fitzpatrick, Kaplan '11] .
- For a CFT$_3$ operator $\phi(x)$ with $\Delta_\phi = 1$,

$$\langle \phi(x_1)\phi(x_2)\phi(x_3)\phi(x_4)\rangle_{\text{conn}} = \frac{1}{x_{12}^2 x_{34}^2} g(U, V)$$

go to Mellin space

$$g(U, V) = \int \frac{ds\, dt}{(4\pi i)^2} U^{t/2} V^{(u-2)/2} \Gamma^2\left(1 - \frac{s}{2}\right) \Gamma^2\left(1 - \frac{t}{2}\right) \Gamma^2\left(1 - \frac{u}{2}\right) M(s, t)$$

where $s + t + u = 4$ and $U = \frac{x_{12}^2 x_{34}^2}{x_{13}^2 x_{24}^2}$, $V = \frac{x_{14}^2 x_{23}^2}{x_{13}^2 x_{24}^2}$.

- From the large s, t limit of $M(s, t)$ one can extract 4d scattering amplitude $\mathcal{A}(s, t)$ [Penedones '10; Fitzpatrick, Kaplan '11] .

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- To obtain scattering amplitude of graviton + superpartners in M-theory, look at **stress tensor multiplet** in ABJM theory (ABJM theory is a 3d $\mathcal{N} = 8$ **SCFT**, and so it has $\mathfrak{so}(8)_R$ R-symmetry):

	dimension	spin	$\mathfrak{so}(8)_R$	couples to
focus on this $\longrightarrow$	1	0	$\mathbf{35}_c$	scalars
	3/2	1/2	$\mathbf{56}_v$	gravi-photinos
	2	0	$\mathbf{35}_s$	pseudo-scalars
	2	1	$\mathbf{28}$	gravi-photons
	5/2	3/2	$\mathbf{8}_v$	gravitinos
	3	2	$\mathbf{1}$	graviton

- Task: find the Mellin amplitude $M(s,t)$ corresponding to $\langle S_{IJ} S_{KL} S_{MN} S_{PQ} \rangle$ by solving superconformal Ward identity [Dolan, Gallot, Sokatchev '04] order by order in $\ell_p^2 \propto N^{-1/3} \propto c_T^{-2/9}$.

  Here, $\langle T_{\mu\nu} T_{\rho\sigma} \rangle \propto c_T \propto N^{3/2}$.

- Require: 1) at order ℓ_p^{2k}, $M(s,t)$ should not grow faster than $(k+1)$st power of s, t, u;
2) right analytic properties to correspond to a bulk tree-level Witten diagram.
- Number of such solutions to Ward identity is:

degree in s, t, u	1	2	3	4	5	6	7	...
11D vertex	R			R^4		D^4R^4	D^6R^4	...
scaling	c_T^{-1}			$c_T^{-\frac{5}{3}}$		$(0\times)c_T^{-\frac{19}{9}}$	$c_T^{-\frac{7}{3}}$	
# of params	1			2		3	4	...

(degree 1 in [Zhou '18]); degree ≥ 2 in [Chester, SSP, Yin '18] .)
So:
- To determine $M(s,t)$ to order $1/c_T$ we should compute **one** CFT quantity.
- To determine $M(s,t)$ to order $1/c_T^{5/3}$ we should compute **two** CFT quantities.

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CFT quantities

- The CFT quantities can be, for instance, squared OPE coefficients appearing in the superconformal block decomposition. Schematically,

$$\langle S_{IJ} S_{KL} S_{MN} S_{PQ} \rangle = \frac{1}{x_{12}^2 x_{34}^2} \sum_{\mathcal{N}=8 \text{ supermultiplets } \mathcal{M}} \lambda_{\mathcal{M}}^2 \mathcal{G}_{\mathcal{M}}(U, V)\,.$$

  ($\mathcal{M}$ is a superconformal multiplet appearing in the $S \times S$ OPE.)

- $T_{\mu\nu}$ Ward identity gives $\lambda^2_{\text{stress}} = 256/c_T$. Using SUSY tricks, one can compute [Agmon, Chester, SSP '17] :

$$\lambda^2_{B,2} = \frac{32}{3} - \frac{1024(4\pi^2 - 15)}{9\pi^2} \frac{1}{c_T} + 40960 \left(\frac{2}{9\pi^8}\right)^{\frac{1}{3}} \frac{1}{c_T^{5/3}} + \cdots$$

  where "stress" is the stress tensor multiplet, and "$(B, 2)$" is a $1/4$-BPS mutliplet appearing in the OPE $S \times S$.

Precision test of AdS/CFT

- Using these two expressions, we determined $M(s, t)$ to order $1/c_T^{5/3}$.
- The flat space limit implies $f_{R^4}(s, t) = \frac{stu}{3 \cdot 2^7}$, as expected.
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OPE coefficients from SUSic localization

- It is hard to calculate correlation functions at separated points using SUSic localization. See however [Gerkchovitz, Gomis, Ishtiaque, Karasik, Komargodski, SSP '16; Dedushenko, SSP, Yacoby '16] .

How were c_T and $\lambda^2_{(B,2)}$ computed?

- From derivatives of the S^3 partition function with respect to an $\mathcal{N} = 4$-preserving mass parameter m, which can be computed using supersymmetric localization.
- (For c_T, see also [Chester, Lee, SSP, Yacoby '14] for another method based on [Closset, Dumitrescu, Festuccia, Komargodski, Seiberg '12] .)

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Mass-deformed S^3 partition function

- For an $\mathcal{N} = 4$-preserving mass deformation of ABJM theory, $Z_{S^3}(m)$ is [Kapustin, Willett, Yaakov '09] :

$$Z_{S^3}(m) = \int d^N\lambda \, d^N\mu \, e^{ik\sum_i(\lambda_i^2-\mu_i^2)} \frac{\prod_{i<j} \sinh^2(\lambda_i - \lambda_j) \sinh^2(\mu_i - \mu_j)}{\prod_{i,j} \cosh(\lambda_i - \mu_j + m) \cosh(\lambda_i - \mu_j)}$$

- Small N: can evaluate integral exactly.
- Large N: rewrite $Z_{S^3}(m)$ as the partition function of non-interacting Fermi gas of N particles with [Marino, Putrov '11; Nosaka '15]

$$U(x) = \log(2\cosh x) - mx \,, \qquad T(p) = \log(2\cosh p) \,.$$

Resummed perturbative expansion [Nosaka '15] :

$$Z_{S^3}(m) \sim \mathrm{Ai}\left(f_1(m)N - f_2(m)\right)$$

for some known functions $f_1(m)$ and $f_2(m)$. ($\log Z \propto N^{3/2}$)

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Topological sector

- 3d $\mathcal{N} = 4$ SCFTs have a 1d topological sector [Beem, Lemos, Liendo, Peelaers, Rastelli, van Rees '13; Chester, Lee, SSP, Yacoby '14; Dedushenko, SSP, Yacoby '16] defined on a line $(0, 0, x)$ in $\mathbb{R}^3$.
- $\langle \mathcal{O}_1(x_1) \dots \mathcal{O}_n(x_n) \rangle$ depends only on the ordering of x_i on the line.
- Ops in 1d are 3d 1/2-BPS operators $\mathcal{O}(\vec{x})$ placed at $\vec{x} = (0, 0, x)$ and contracted with x-dependent R-symmetry polarizations.
- The operators $\mathcal{O}(x)$ are in the cohomology of a supercharge $\mathbb{Q} =$ "$Q + S$" cohomology s.t. translations in x are $\mathbb{Q}$-exact.
- The topological sector is defined either on a line in flat space or on a great circle of S^3.
- In ABJM, construct 1d operators $S_\alpha(x)$ from S_{IJ}, $\alpha = 1, 2, 3$. Their 2-pt function depends on c_T; their 4-pt function depends on c_T and $\lambda^2_{(B,2)}$.

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From $Z_{S^3}(m)$ to OPE coefficients

- One can show $Z_{S^3}(m) = Z_{S^1}(m)$, and so derivatives of the Z_{S^3} w.r.t. m corresponds to integrated correlators in the 1d theory.
- From 2 derivatives of Z_{S^3} w.r.t. m we can extract c_T.
- From 4 derivatives of Z_{S^3} w.r.t. m we can extract $\lambda^2_{(B,2)}$.
- **So the (resummed) perturbative expansion of c_T, $\lambda^2_{B,2}$ can be written in terms of derivatives of the Airy function!**
- Eliminating N gives

$$\lambda^2_{B,2} = \frac{32}{3} - \frac{1024(4\pi^2 - 15)}{9\pi^2}\frac{1}{c_T} + 40960\left(\frac{2}{9\pi^8}\right)^{\frac{1}{3}}\frac{1}{c_T^{5/3}} + \cdots$$

- (Tangent: For 2d bulk dual of the 1d topological sector of ABJM theory, see [Mezei, SSP, Wang '17] . The 1d theory is exactly solvable, and its 2d bulk dual is 2d YM.)

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Beyond f_{R^4}?

- Can one go beyond reconstructing f_{R^4}?
- More SUSic localization results for ABJM theory are available: Z_{S^3} as a function of three real mass parameters; partition function on a squashed sphere, etc.
  - Cannot use the 1d topological sector in this case, but it is very likely that this extra data will show $f_{D^4 R^4} = 0$ and maybe even determine $f_{D^6 R^4}$. (Work in progress with D. Binder and S. Chester.)
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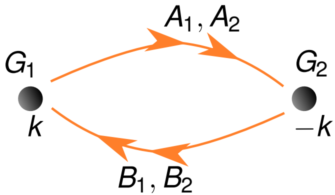
  - Cannot use the 1d topological sector in this case, but it is very likely that this extra data will show $f_{D^4 R^4} = 0$ and maybe even determine $f_{D^6 R^4}$. (Work in progress with D. Binder and S. Chester.)
- Another approach: conformal bootstrap.
  - Generally, we obtain bounds on various quantities.
  - If the bounds are saturated, then we can solve for the CFT data.

Known $\mathcal{N} = 8$ SCFTs

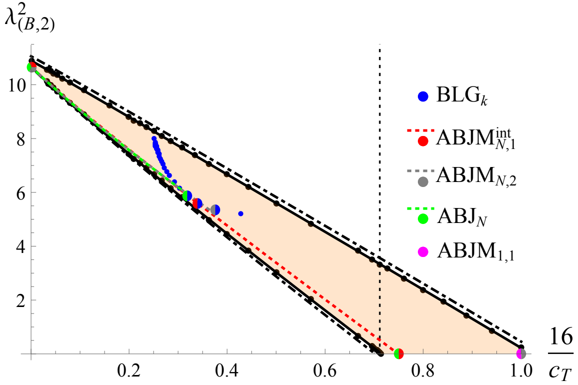
A few families of $\mathcal{N} = 8$ SCFTs:



- With holographic duals:
  - $\text{ABJM}_{N,1}$: $U(N)_1 \times U(N)_{-1} \quad \longleftrightarrow \quad AdS_4 \times S^7$.
  - $\text{ABJM}_{N,2}$: $U(N)_2 \times U(N)_{-2} \quad \longleftrightarrow \quad AdS_4 \times S^7/\mathbb{Z}_2$.
  - $\text{ABJ}_{N,2}$: $U(N)_2 \times U(N+1)_{-2} \quad \longleftrightarrow \quad AdS_4 \times S^7/\mathbb{Z}_2$.
- Without known holographic duals:
  - BLG_k: $SU(2)_k \times SU(2)_{-k}$.

Bootstrap bounds [Agmon, Chester, SSP '17]

- Bounds from conformal bootstrap applying to all $\mathcal{N} = 8$ SCFTs.



- SUGRA (leading large c_T) saturates bootstrap bounds.
- Conjecture: $\text{ABJM}_{N,1}$ or $\text{ABJM}_{N,2}$ or $\text{ABJ}_{N,2}$ saturate bound at all N in the limit of infinite precision.

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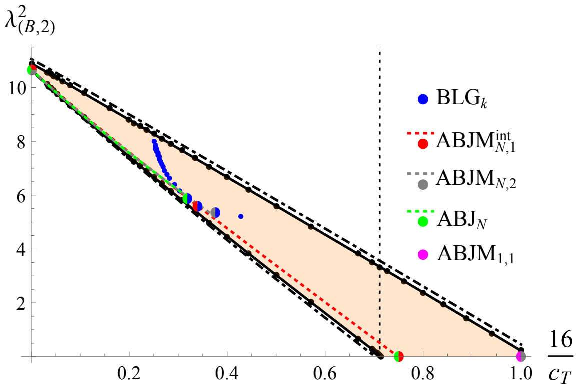

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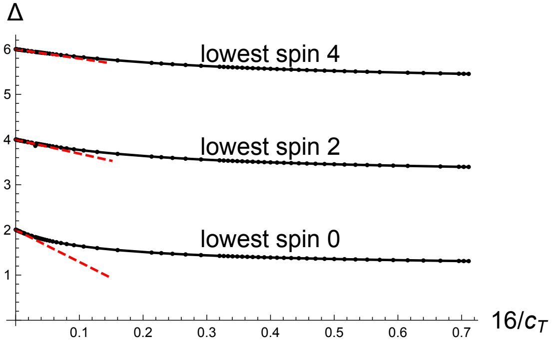
Bound saturation $\Longrightarrow$ read off CFT data

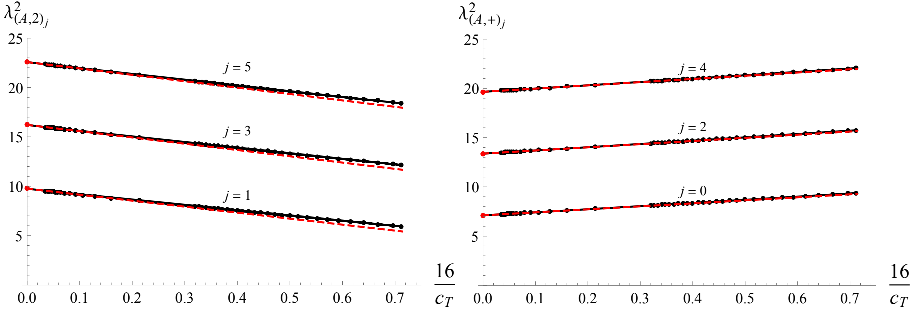
- On the boundary of the bootstrap bounds, the solution to crossing should be unique $\Longrightarrow$ can find $\langle S_{IJ} S_{KL} S_{MN} S_{PQ} \rangle$ and solve for the spectrum !! [Agmon, Chester, SSP '17]



Red lines are leading SUGRA **tree level** results [Zhou '17; Chester '18] .
Lowest operators have the form $S_{IJ} \partial_{\mu_1} \cdots \partial_{\mu_\ell} S^{IJ}$.

$\lambda^2_{(A,2)_j}$ and $\lambda^2_{(A,+)_j}$ from extremal functional

Semishort $(A,2)_j$ and $(A,+)_j$ OPE coefficients for low spin j in terms of $\frac{16}{c_T}$ from extremal functional:



- Red line is tree level SUGRA result [Chester '18] .
- $\lambda^2_{(A,+)_j}$ appears close to linear in $16/c_T$.
- More precision needed.

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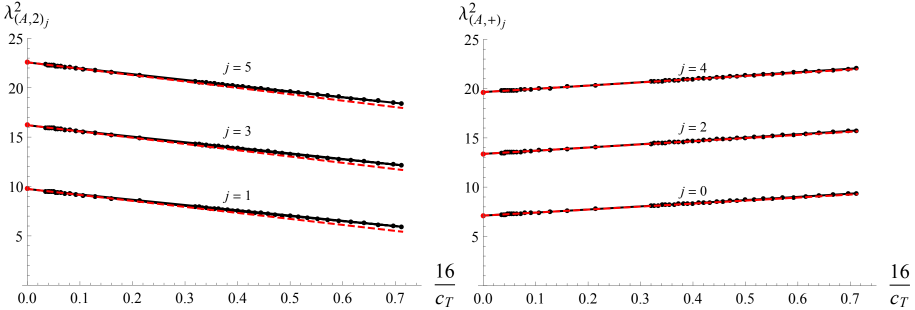

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Conclusion

- Can compute OPE coefficients in $\mathcal{N} = 8$ SCFTs with Lagrangian descriptions using supersymmetric localization.
- For ABJM theory, we can reproduce the $f_{R^4}(s,t) = \frac{stu}{3 \cdot 2^7}$ term in the flat space 4-graviton scattering amplitude.
- Bootstrap bounds are almost saturated by $\mathcal{N} = 8$ SCFTs with holographic duals.

For the future:

- Generalize to other dimensions, other 4-point function, less SUSY. (See [Chester, Perlmutter '18] on 6d as well as Shai Chester's talk & poster.)
- Study other SCFTs from which one can compute scattering amplitudes of gauge bosons on branes. (?)
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