Coulomb branches of 4d QFTs

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arXiv:1504.04814, 1601.00011, 1602.02764, 1609.04404, 1611.08602, 1704.05110, 1801.01122, 1801.06554, 1804.03152, & to appear...

Goal: systematically classify CB geometries to constrain possible 4d $\mathcal{N}=2$ QFTs

Main problem: what kinds of singularities in the CB geometry are allowed?

Coulomb branch (CB) basics

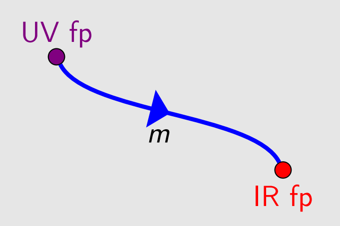
- CB is a component of the moduli space of vacua of a 4d $\mathcal{N} = 2$ supersymmetric (non-gravitational) QFT with massless $\mathcal{N} = 2$ $U(1)$ gauge fields.
- The kinetic terms of the effective action of the massless complex scalars, Φ, in the $U(1)$ gauge multiplets gives the CB a metric structure. At its metrically smooth points $\mathcal{N} = 2$ supersymmetry gives it a special Kähler (SK) structure. At these points $r := \dim_{\mathbb{C}}(\text{CB})$ is its **rank**, and the low energy gauge group is $U(1)^r$.
- Each point is a distinct vacuum coordinatized by the vevs $u = \langle \Phi \rangle$ and nearby vacua are at finite distances: the CB is metrically complete and its metric topology coincides with its continuous topology.
- The metric is non-analytic ("singular") at vacua where charged states become massless. There need not be a singularity in the complex or topological manifold structures on the CB at the metric singularities.

Some assumptions

CB geometry at metric infinity (large vevs) reflect UV behavior of QFT.

Assume: large-distance asymptotics of CB are scale-invariant.

So these CBs can come from QFTs which flow from a UV f.p., which we assume is a SCFT.



$\Rightarrow$ Our classification of CB geometries will constrain $\mathcal{N} = 2$ SCFTs.

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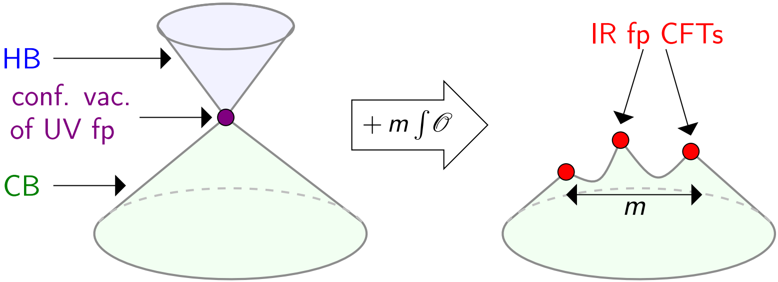

$\Rightarrow$ CB not lifted by local deformations, CB gives "map" of RG flow.
Unique SCFT vacuum $\Rightarrow$ CB connected.

Some assumptions

Assume: large-distance asymptotics of CB are scale-invariant.

Metric singularities occur at vanishing central charge, $Z_q(u) = 0$, but still...

Assume: singular locus is a complex-analytic set.

$\Rightarrow$ singular locus closed, not a natural boundary

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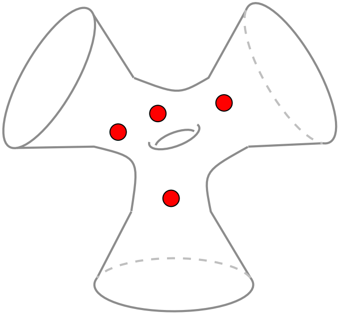
Still many possibilities...



Some assumptions

Assume: large-distance asymptotics of CB are scale-invariant.

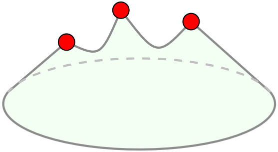
Assume: singular locus is a complex-analytic set.

More assumptions just to get started: (will try to lift them later)

Assume: $\text{rank(CB)} = 1$.

Assume: CB "planar": $\text{CB} \simeq_{\mathbb{C}} \mathbb{C}$.

Assume: No interacting "rank-0" SCFTs.

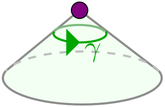


$\Rightarrow$ finite number of singular points

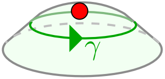
Scaling forms of singularities

SK structure and unitarity ($\Delta(u) \geq 1$) $\Rightarrow$

- Possible singularities classified by conjugacy class of EM duality $SL(2,\mathbb{Z})$ monodromy $[M_\gamma]$.
- Metric can have conical singularity with opening angle $2\pi/\Delta(u)$ plus logarithmic corrections.



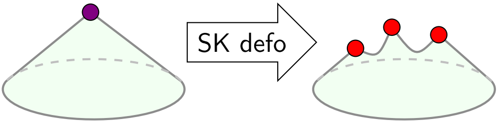
	I_0	II	III	IV	I_0^*	IV^*	III^*	II^*
$\Delta(u)$	1	6/5	4/3	3/2	2	3	4	6
$[M_\gamma]$	I	$(ST)^{-1}$	S^{-1}	$(ST)^{-2}$	$-I$	$(ST)^2$	S	ST



	I_n	I_n^*
$\Delta(u)$	1	2
$[M_\gamma]$	T^n	$-T^n$

($n > 0$, not scale-invariant)

SK deformations



- Complex deformations of scale invariant geometries which preserve an SK strucure are characterized by the patterns of splitting of the UV singularity which preserve the total EM monodromy $\Rightarrow$ many possible patterns.
- Exists [SW'94,MN'96] *maximal deformations* $X \to \{I_1{}^n\}$, which are families of CB geometries depending on complex parameters $\mathbf{m} \in \mathscr{M}_{\text{max}} \sim \mathbb{C}^f$
- All other deformations are restrictions of these maximal families to certain subspaces $\mathscr{M} \subset \mathscr{M}_{\text{max}}$ [Caorsi Cecotti 1803.00531]
- **But:** the IR physics of sub-maximal deformations can be qualitatively different from that of the maximal deformation, so we should distinguish them as inequivalent theories!

"Frozen" IR fixed points

- Example:
  - I_n singularity arises from an IR free $U(1)$ gauge theory with n $q = \pm 1$ massless hypermultiplets.
  - This has $n{-}1$ relative masses = deformations splitting $I_n \to \{{I_1}^n\}$.
  - But I_n singularity also arises from $U(1)$ with a single $q = \pm\sqrt{n}$ hyper.
  - This doesn't have a relative mass deformation $\Rightarrow$ "**frozen singularity**".
- This is familiar already in the simplest lagrangian example:
  - $SU(2)$ gauge theory with $N_f = 4 \Rightarrow I_0^* \to \{{I_1}^6\}$
  - $SU(2)$ with $N_{adj} = 1 \Rightarrow I_0^* \to \{{I_1}^2, I_4\}$, found by restricting parameters
- Constrained by Dirac quantization: the set $\{\sqrt{n_i}\}$ commensurate $\Rightarrow$ ...

28 possible rank-1 planar CB geometries for SCFTs

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- NB: in all cases in our classification, Γ is actually a real crystallographic reflection group, i.e., a Weyl group.

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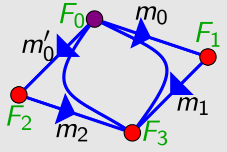

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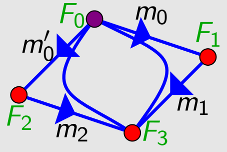

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For one of the 28 geometries, this constraint has no solutions: it is ruled out as inconsistent

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- Conversely: the success of our classification could be evidence for their non-existence.

Comparison to rank-1 SCFTs

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So, AFAIK:

SCFT $\rightarrow$ CB geometry map is neither 1-to-1 nor onto.

CB chiral rings

The assumption that CB $\simeq_{\mathbb{C}} \mathbb{C}$ is equivalent to the chiral ring, $\mathscr{O}_{\text{CB}}$, of the CB operators in the UV SCFT being freely generated: $\mathscr{O}_{\text{CB}} = \mathbb{C}[u]$.

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A singularity in the CB complex structure then gives a non-freely-generated CB chiral ring. For example,

$$\mathscr{O}_{\text{CB}} = \mathbb{C}[a, b]/\langle a^3 - b^2 \rangle.$$

In terms of the normalizing coordinate,

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Now $a = \langle \Phi_a \rangle$, $b = \langle \Phi_b \rangle$ are vevs of SCFT local operators, but u is not:

Can have $\Delta(u) < 1$ without violating the SCFT unitarity bound.

Non-freely-generated CB chiral rings?

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Ff such theories exist, they form a separate set under RG flows.

Rank > 1?

Much less is known. **2 notable results:**
[PCA,Martone 1801.06554, 1804.03152]

- Can construct (lagrangian, $\mathcal{N} = 4$!) SCFTs with CB complex singularities by discrete gauging.
  - CB chiral ring is not freely-generated.
  - In these examples, CBs are normal varieties (unlike rank-1 singular varieties), and no negative curvatures.
  - Also: [Bourget, Pini, Rodriguez-Gómez 1804.00118].
- Can compute the (finite, rational) set of allowed CB scaling dimensions for any rank $r \geq 2$.
  - Assumes CB $\simeq_{\mathbb{C}} \mathbb{C}^r$.
  - Also: [Caorsi, Cecotti 1801.04542].

In progress: exploring the tight structure tying stratified topology of the locus of metric singularities to the set of EM duality monodromies around these singularities.

End

Thanks!