From discrete surface theory to architecture and back

Wayne Rossman
Kobe University

One aspect of creating city environments that are pleasant for the inhabitants is building and pavilion architecture, and recent technology has allowed the possibility of curved faces on these structures. These surfaces are often comprised of many flat pieces connected along edges, creating a grid, or net. Designing such structures that are both sturdy, enough to withstand powerful earthquakes for example, and maximally appealing to the eye, is a unique challenge that, in spite of the surfaces' locally discretized shapes, requires (perhaps surprisingly) a deep knowledge of the differential geometry of surfaces. Thus, we will discuss the foundations of discrete surface theory that underlie the corresponding ideas in architecture.

More specifically about the mathematics involved, consider this
example: When building a curved surface on a building comprised of many flat glass planes, it is advantageous to use quadrilateral faces rather than triangular ones, producing less wasted material (when cutting the glass) and also a lighter (and hence stronger) structure.
It is also advantageous to build with two parallel surfaces, to greatly increase strength, and to better insulate the building from the outside weather, thus saving on heating and air-conditioning expenses. However, when installing both surfaces, along with supporting beams along the edges of the quadrilaterals and supporting faces between corresponding edges of the two surfaces, one encounters difficult problems of a geometric nature. Notions from differential geometry such as normal fields, curvature, torsion and metrics come into play in an essential way.

Furthermore, one can enhance the natural beauty of the curved surfaces by introducing coordinate chart concepts from differential geometry. For example, if the edges between faces are laid out to imitate coordinates for smooth surfaces that are both conformal (surface tension minimizing) and determined by principal curvature directions (directions of strongest bending in the surfaces), the resulting grid will have greater aesthetic appeal.

While the mathematical concepts are usefully applied to architectural problems, likewise architecture motivates interest in discrete differential geometry as a purely mathematical field. Toward the end of the talk, we shall touch upon topics in the field that do not necessarily have applications outside of pure mathematics, by considering discrete surfaces in non-Euclidean spaces, transformation theory for discrete surfaces, and notions of singularities on discrete surfaces.

This talk will have clear relations with the talk by Helmut Pottmann at this GEMS meeting.