de Sitter Holography and Entanglement Entropy

Based on:

*ArXiv 1804.08623 with Xi Dong & Gonzalo Torroba

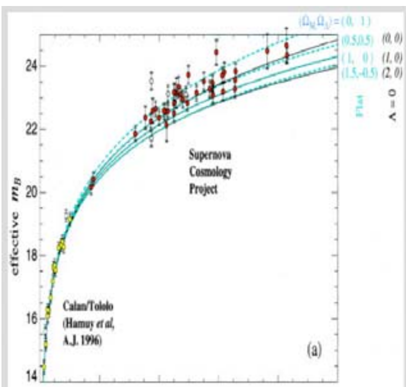
*dS/dS and FRW/FRW dualities:
2004-2011
with Alishahiha, Dong, Horn, Karch, Matsuura, Tong, Torroba

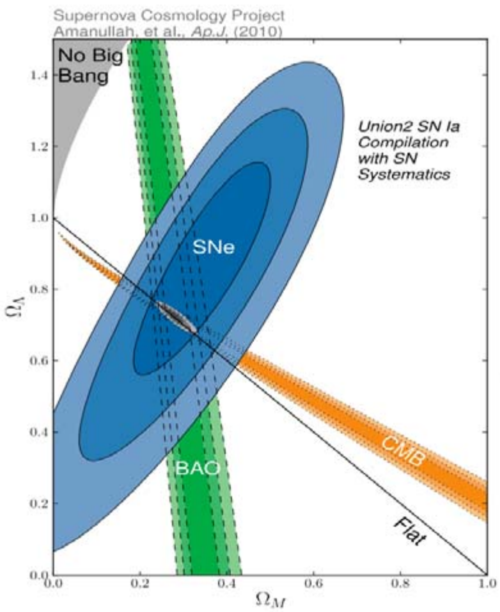
*Somewhat related refs: Miyaji,Takayanagi, Sato, Narayan, Mozaffar/Mollabashi, Nomura/Rath/Salzetta,...

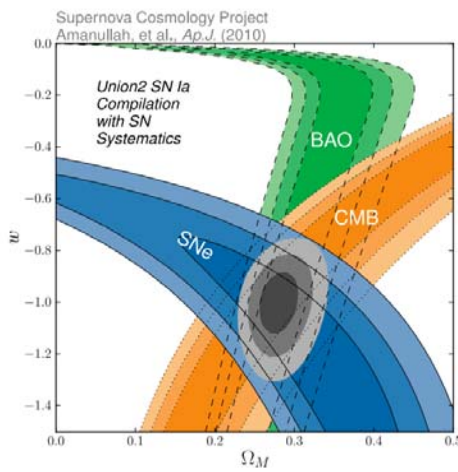
Accelerated expansion dominates

Observationally:

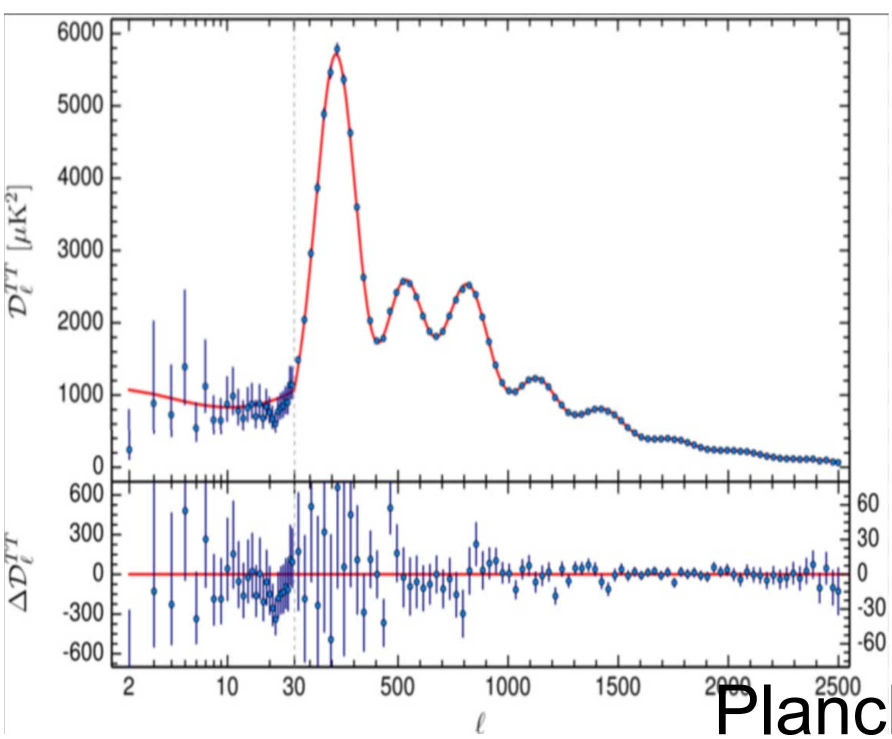
Late Universe

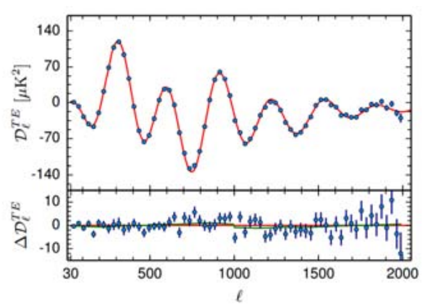


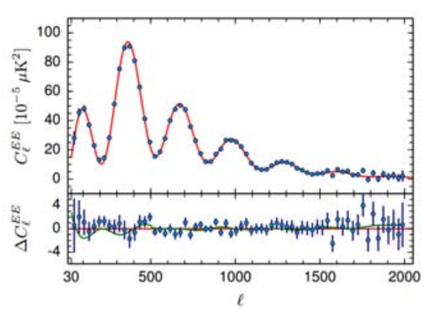




Early Universe: inflation fits well

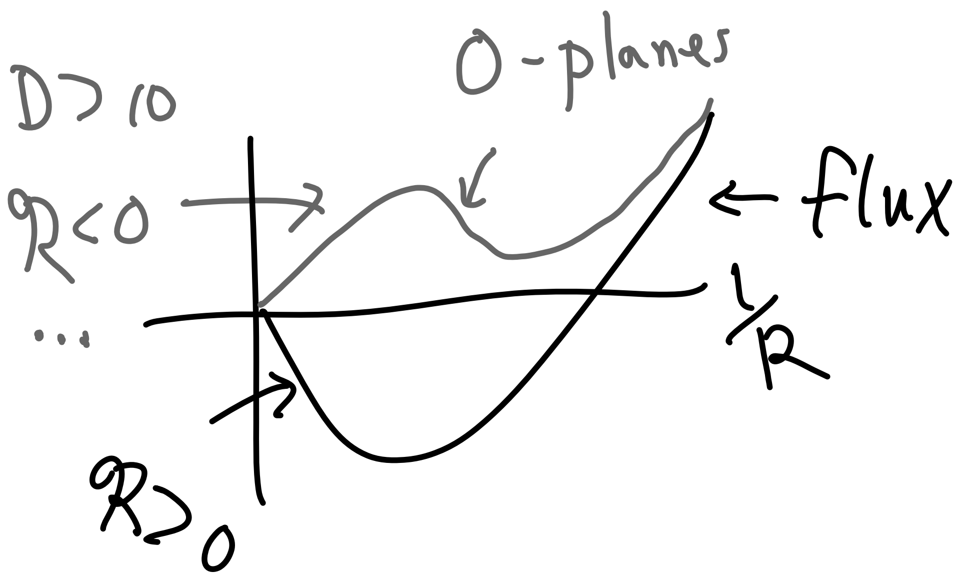






Planck

Positive potential V(scalars) dominates String-Theoretically, with controlled negative sources (orientifolds & cousins) plausibly producing metastable dips in otherwise steep directions

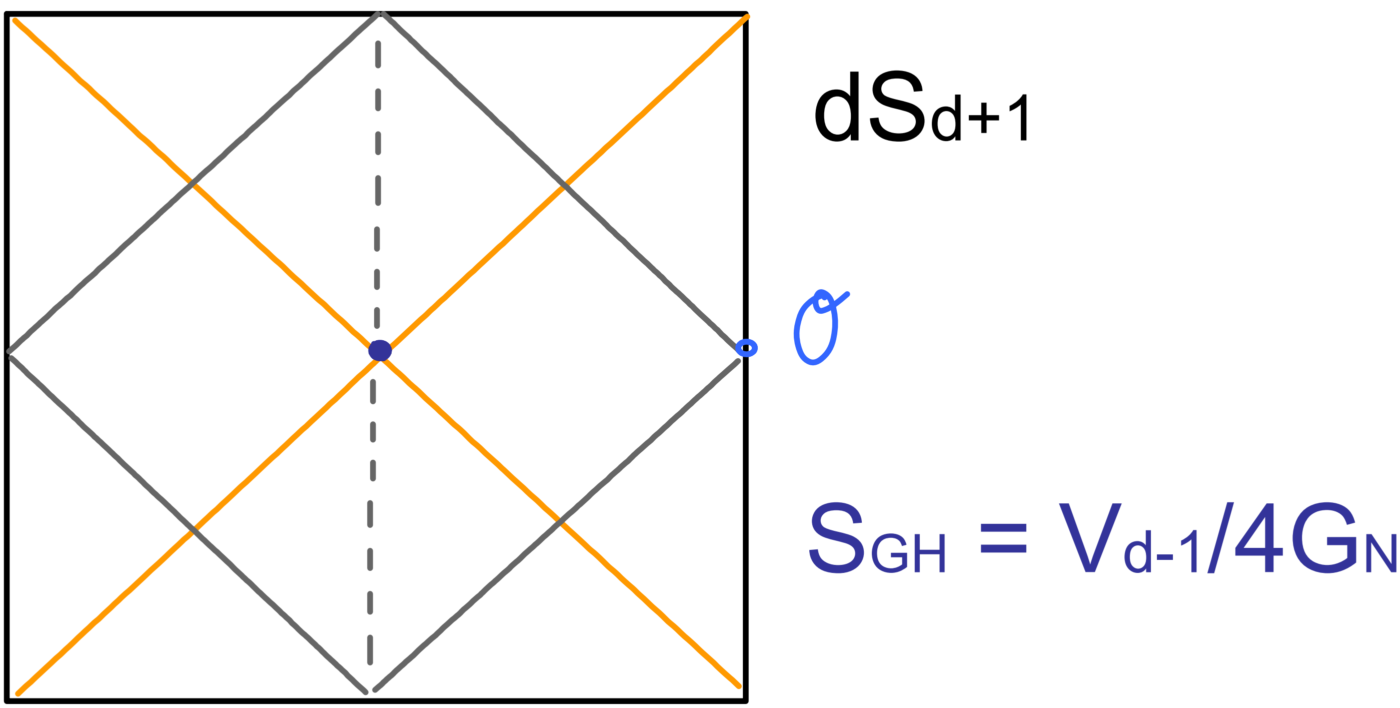


+other sources to fully stabilize

Various mechanisms:
Supercritical (ES 2001+...), GKP '01/KKLT'03, LARGE Volume, Riemann Surfaces,...
Explicit example (2+1 bulk dimensions):
Dong, ES, Torroba 2010.

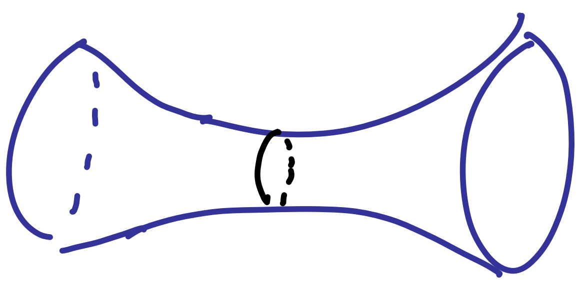
Thought-experimentally:

Gibbons/Hawking dS entropy demands QG interpretation



More generally, we need a more complete QG framework for the V>0 landscape.

AdS/CFT leads to emergent spacetime from gauge theory description. Many aspects to its dictionary. One recent focus is QI

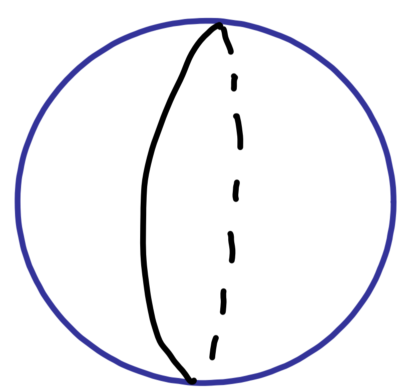


TFD state of two CFTs (entangled at thermal scale) dual to joined spacetime

$$|\Psi\rangle = \sum_n e^{-\beta E_n/2} |n\rangle |n\rangle \, .$$

Find analogous statement for dS, with concrete holographic interpretation of $S_{G\text{-}H}$.

Entangled state of 2 interacting cut-off CFTs (at cut-off scale).

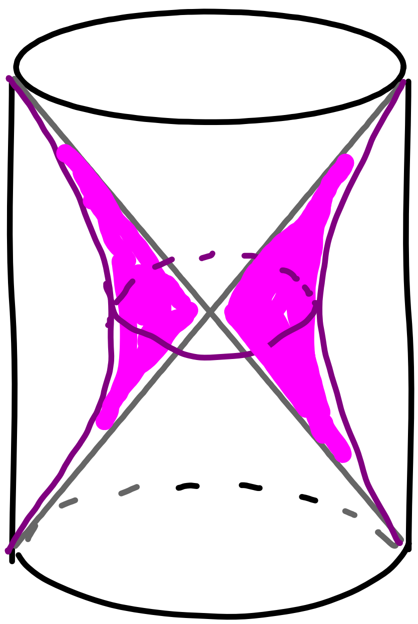


dS_{d+1} = Two coupled cutoff d-dimensional CFTs, constrained by residual d-dimensional gravity (on approximate dS_d geometry)

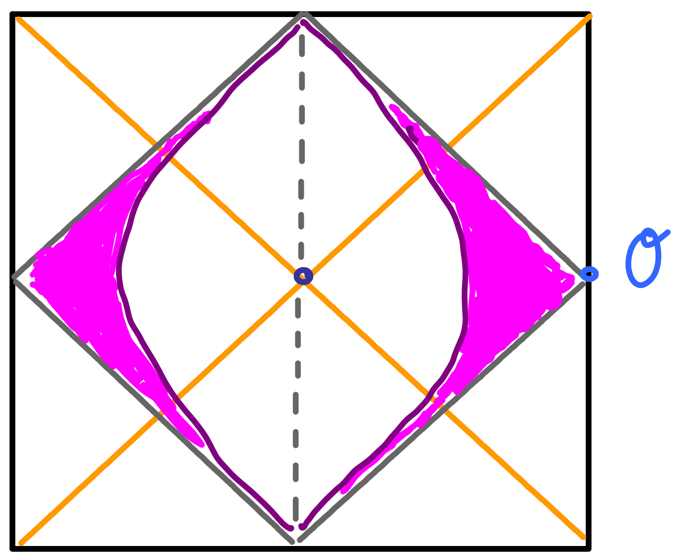
This follows from 2+1 independent arguments, macroscopic and microscopic. They agree <u>because</u> of the meta-stability (no hard cosmological constant in string theory).

Macroscopic:

$$
\begin{aligned}
ds^2_{(A)dS_{d+1}} &= dw^2 + \sin(h)^2\left(\frac{w}{\ell_{dS}}\right) ds^2_{dS_d} \\
&= dw^2 + \sin(h)^2\left(\frac{w}{\ell_{dS}}\right)\left[-d\tau^2 + \ell^2_{dS}\cosh^2\frac{\tau}{\ell_{dS}}d\Omega^2_{d-1}\right].
\end{aligned}
$$



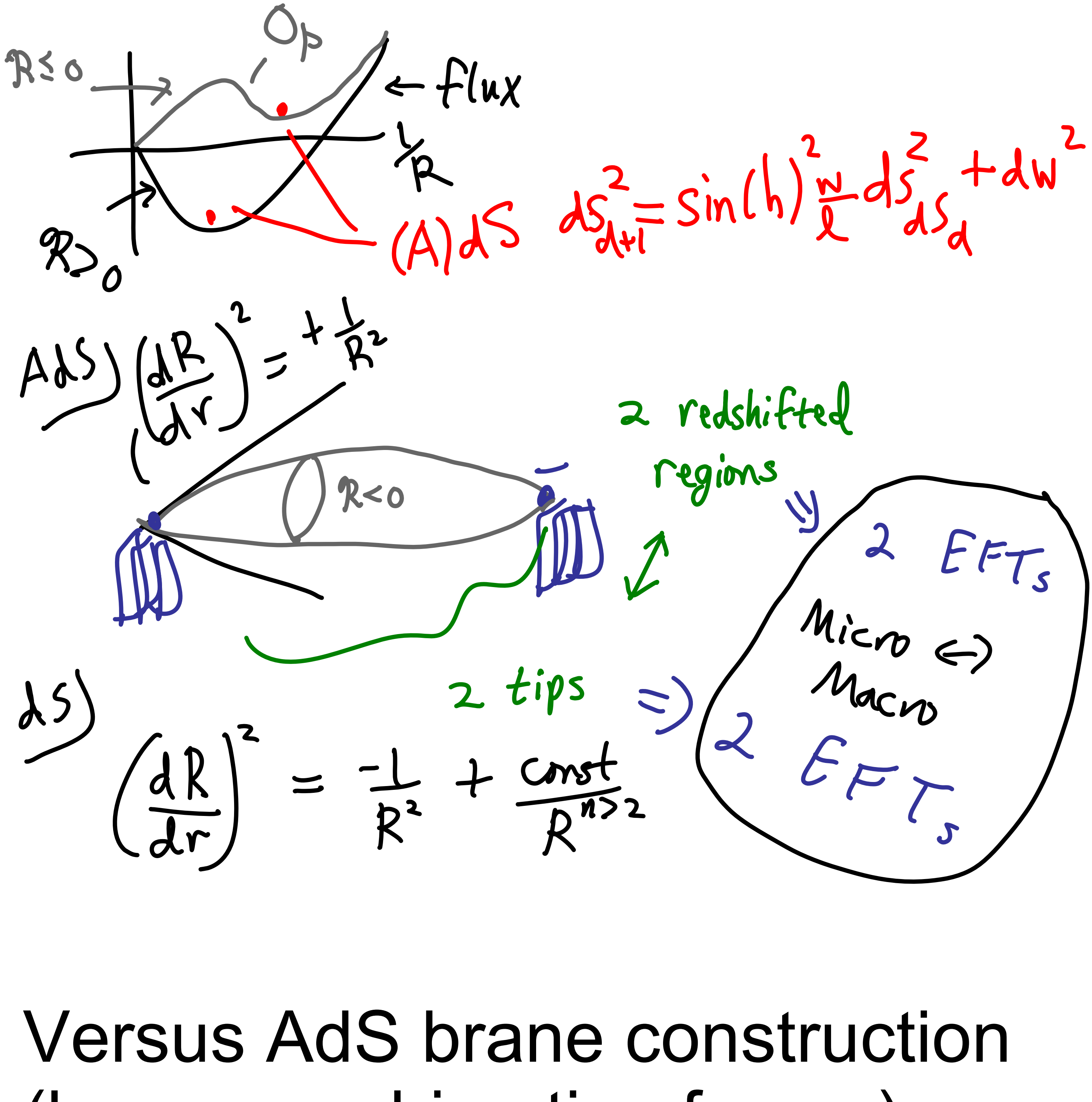
AdS/dS



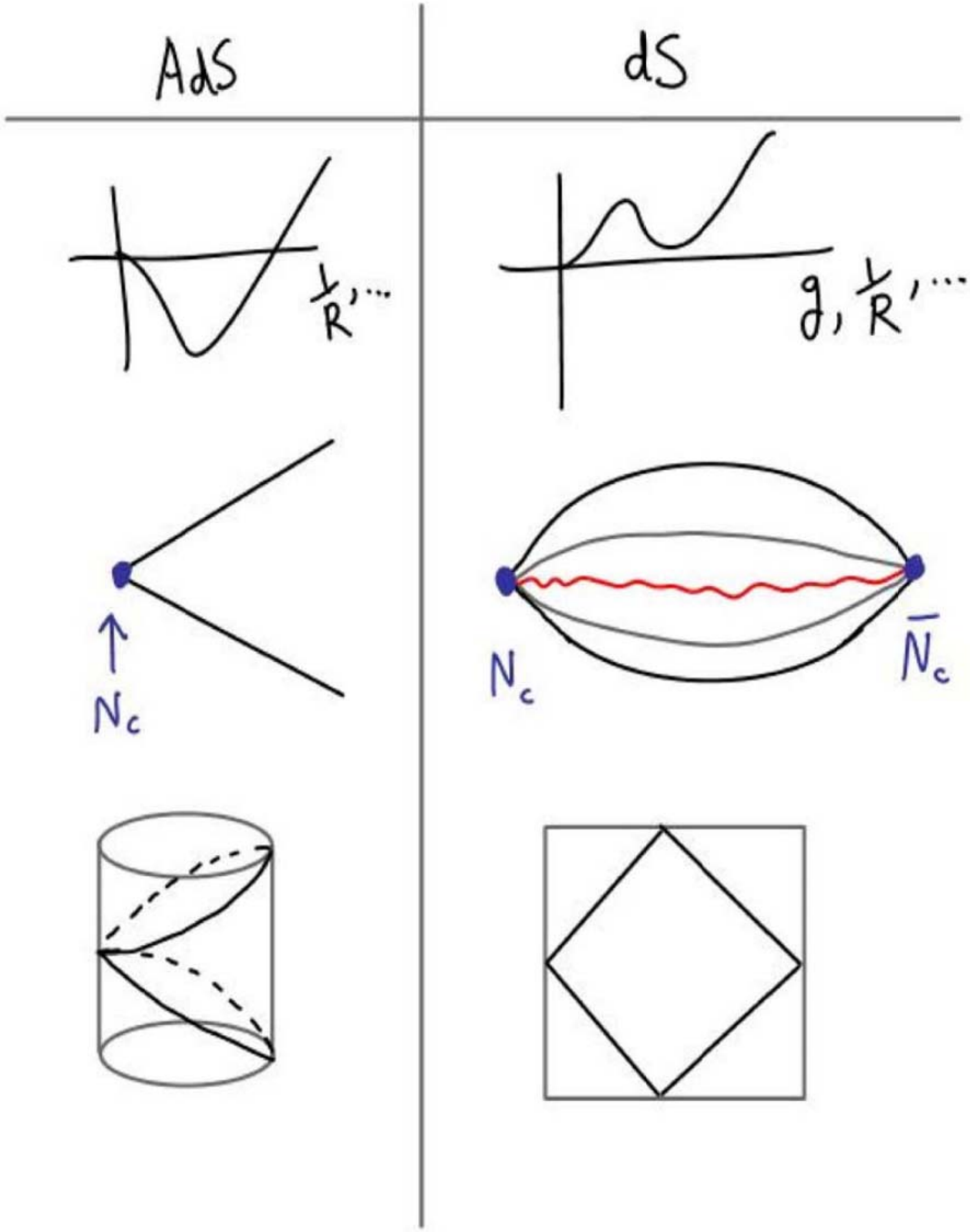
dS/dS (each point is (d-1)-sphere)

2 IR regions, each ~ IR region of AdS/dS

Microscopic: Uplifting AdS/CFT

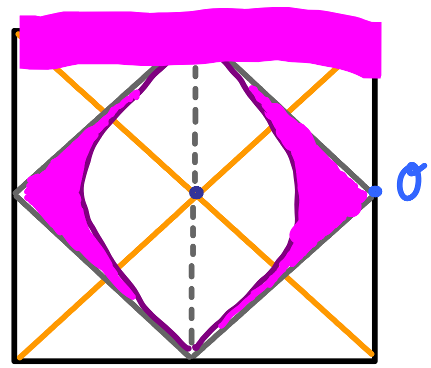


Versus AdS brane construction (branes probing tip of cone).


AdS
dS
$\frac{1}{R}, \ldots$
$g, \frac{1}{R}, \ldots$
N_c
N_c
$\bar{N}_c$

The dS/CFT conjecture gives a third indication of this structure

Strominger; Anninos, Hartman...



$$Z_{CFT} = \Psi(g_{\mu\nu})$$

$$\langle \mathcal{O} \rangle \sim \int D g_{\mu\nu}^{(d)} \Psi^\dagger (g_{\mu\nu}^{(d)}) \mathcal{O} \Psi (g_{\mu\nu}^{(d)})$$

Maldacena; Harlow/Stanford,...

2 CFTs

Symmetries manifest, but bulk unitarity not; restriction on irrelevant operators Shenker.

dS not obtained by analytic continuation of AdS in string theory (would yield complex fluxes). Decays infect future infinity.

dS_{d+1} = Two coupled cutoff d-dimensional CFTs, constrained by residual d-dimensional gravity (on approximate dS_d geometry)

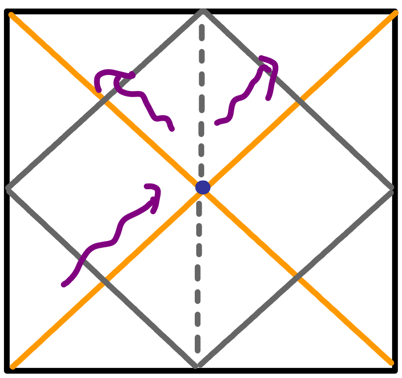
This follows from 2+1 independent arguments, macroscopic and microscopic. They agree <u>because</u> of the meta-stability (no hard cosmological constant in string theory).

pure speculation << dS holography << exact solution

This agreement (despite some remaining ambiguity) seems unlikely to be a coincidence.

Many prior calculations:

*Transmission between the IR regions:



$$T(M', m') = \frac{\sinh^2(\pi m' L)}{\sinh^2(\pi m' L) + \sinh^2 \frac{\pi}{2}\sqrt{4(M'^2 L^2 - 1)}}$$

M', m' related to bulk and d-dim'l `glueball' masses

*Holographic RG: moduli stabilization both for AdS (=> CFT, $\beta_{\text{single-trace}}$=0) and dS. Single-trace couplings also don't flow in dS/dS holographic RG.

*Quasilocal stress tensor => $c_{\text{total}} = 0$ (GR_d)

*...

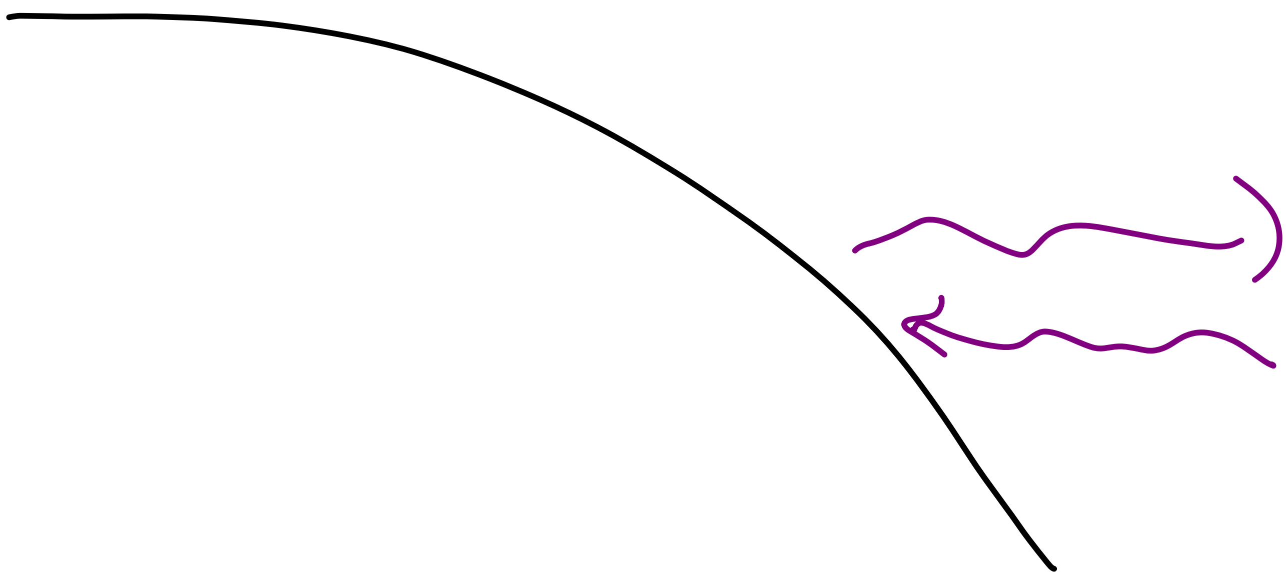
<u>This talk</u>: entanglement structure of the matter sector state, interpretation of dS entropy

Role of GR$_d$: e.g. d=2 Liouville

Hartle/Hawking...Polchinski '89 ... Martinec et al ...

$$\left\{-\frac{\partial^2}{\partial\phi^2} - 2\mathcal{H}_{CFT_1} - 2\mathcal{H}_{CFT_2} - 2\mathcal{H}_{mix} - \cdots - \frac{4}{\kappa^2}e^{\kappa\phi}\right\}\Psi = 0$$

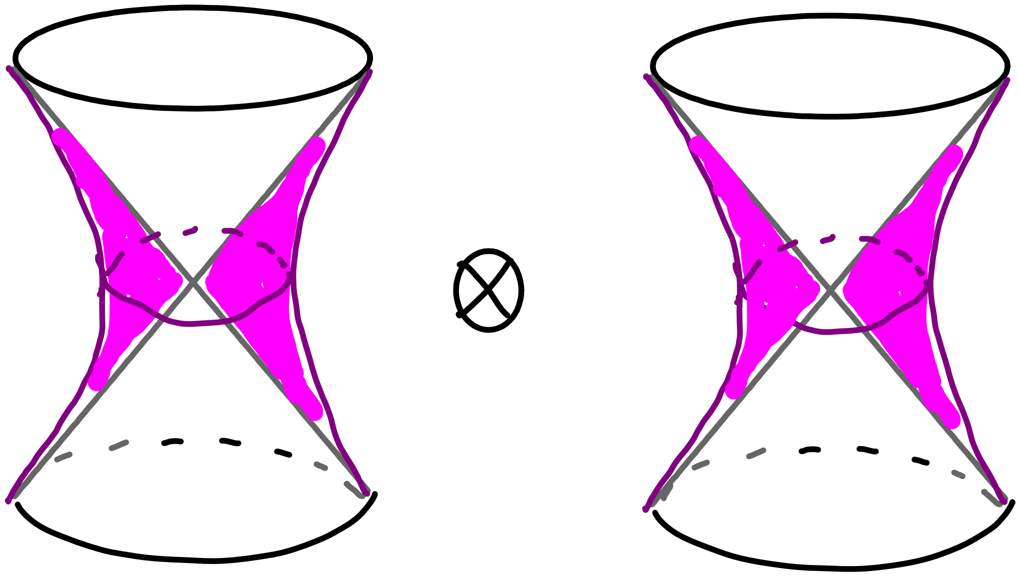
$$g_{\alpha\beta} = e^{\kappa\phi}\eta_{\alpha\beta}$$



Negative matter ground state (Casimir) energy => $\Psi_{\text{Wheeler-deWitt}}$ describes global

Connection to $T\bar{T}$ and JT gravity? cf Dubovsky, Gorbenko, Mirbabayi,...

Cutoff => finite-dimensional matter Hilbert space



Cutoff scale: sinh(w/L)=1. This is the scale of the Hawking-Page transition to large black holes.

$$\log(\dim H) = \mathcal{S} = \frac{1}{4G_N}\,\ell_{AdS}^{d-1}\frac{2\pi^{d/2}}{\Gamma(d/2)}$$

dS3/dS2:

$$\dim(H_{\Delta\leq c/6}) = e^{\pi c/3}$$

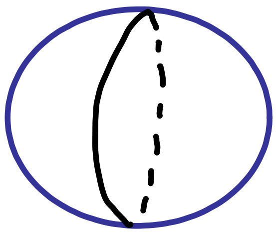
Interactions between the CFTs => ground state (i.e.long-lived dS state) may be highly mixed.

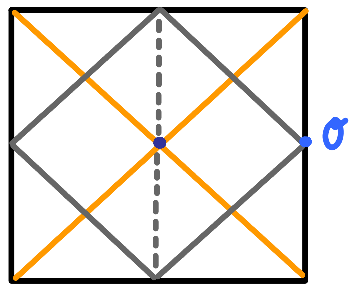
$$
\begin{aligned}
\rho_1 &= \mathrm{Tr}_2(|\Psi\rangle\langle\Psi|) \\
&\simeq \frac{\mathbb{I}}{\dim_{\mathcal{H}}} \; ?
\end{aligned}
$$

Would predict Von Neumann and Renyi entropies are all the same, and equal to $S_{\text{Gibbons-Hawking}}$

$$
\left.
\begin{array}{l}
\mathcal{S} = -\mathrm{Tr}(\rho_1 \log \rho_1) \\[2em]
\mathcal{S}_n = \dfrac{1}{1-n} \log \mathrm{Tr}\, \rho_1^n
\end{array}
\right\} = \log \dim_H
$$

Holographic calculation of Von Neumann and Renyi entropies:





S_{VN} : Extremal surface at $\tau=0$, anchored on UV slice. Volume law (tracing over second matter sector, internal d.o.f.)

$$S_{\text{ent, RT}} = \frac{V_{d-1}}{4G_{d+1}} = S_{\text{Gibbons-Hawking}}$$

dS3/dS2:

$$S_{\text{ent, RT}} = \frac{2\pi\ell_{dS}}{4G_3} = \frac{c\pi}{3} = S_{\text{Gibbons-Hawking}}$$

Holographic Renyi entropies:

Lewkowycz & Maldacena '13, Dong '16.

$$-n^2\partial_n\left(\frac{1}{n}\log \mathrm{Tr}\,\rho_1^n\right) = \quad \tilde{S}_n = \frac{A(C_n)}{4G_{d+1}}$$

Area of `cosmic brane' on Euclidean gravity-side geometry, anchored on UV slice, with deficit angle:

$$T_n = \frac{n-1}{4nG_N} \qquad => \qquad \Delta\phi = 2\pi\frac{n-1}{n}$$

Our Euclidean geometry: (d+1)-sphere/Z_N:

$$|z_1|^2 + |z_2|^2 = \ell_{dS}^2$$

$EdS_{d+1=3}$

$$(z_1, z_2) \simeq (e^{2\pi i/n} z_1, z_2)\,.$$

=> $S_n = S_{VN}$ at large c

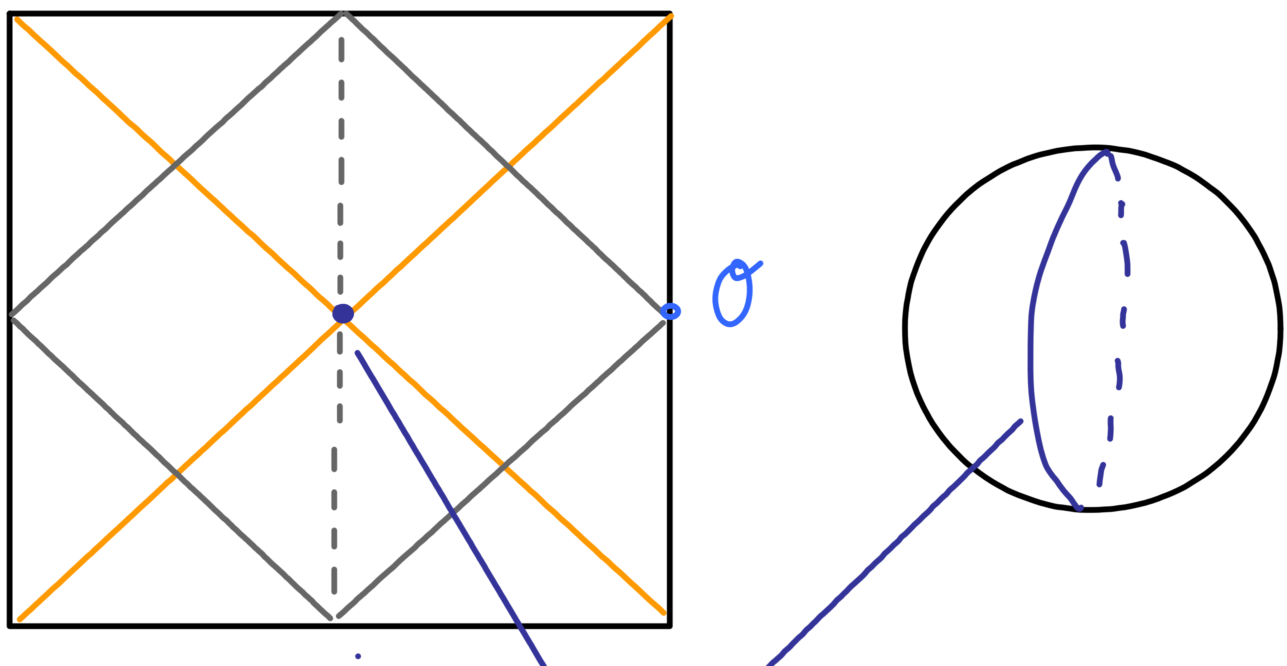
$$z_1 = 0, \qquad |z_2|^2 = \ell_{dS}^2\,.$$

So indeed we have a flat entanglement spectrum (c>>1)

$$\left.\begin{aligned} \mathcal{S} &= -\mathrm{Tr}(\rho_1 \log \rho_1) \\[2em] \mathcal{S}_n &= \frac{1}{1-n} \log \mathrm{Tr}\, \rho_1^n \end{aligned}\right\} = \log \dim_H$$

So the finite Hilbert space we derived earlier, combined with this ~ maximally mixed state, gives the dS entropy as the entanglement entropy between the 2 matter sectors (including the coefficient).

Interpretation of $S_{\text{Gibbons-Hawking}}$:



RT Surface $\frac{A}{4G_N} = S_{RT}$

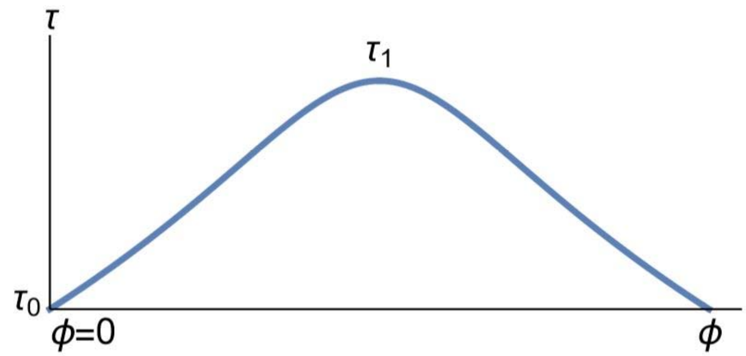
$= S_{vN} = \log \dim H = S^{(O)}_{G-H}$

Observer O cannot interact with 2nd matter sector, traces over it.

Late times τ >0 in dS

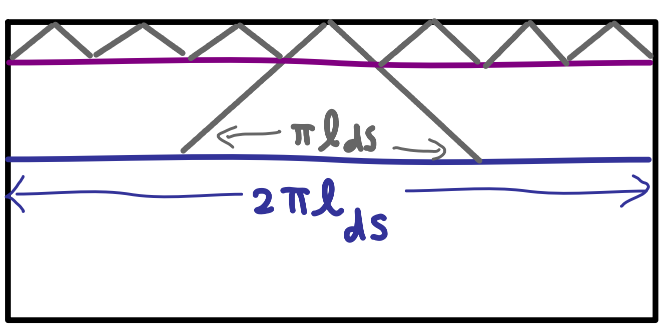
Hubeny/Rangamani/Takayanagi (HRT)

dS_3/dS_2: extremal surface is a geodesic at UV slice.

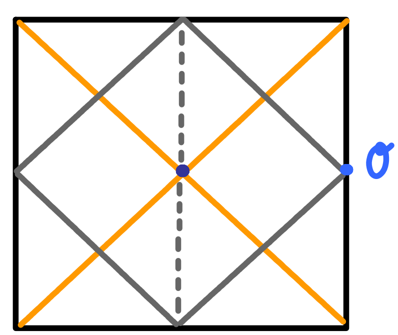


The geodesic becomes complex at late times, when its peak hits future infinity.

S = πc/6 for each such Hubble patch.

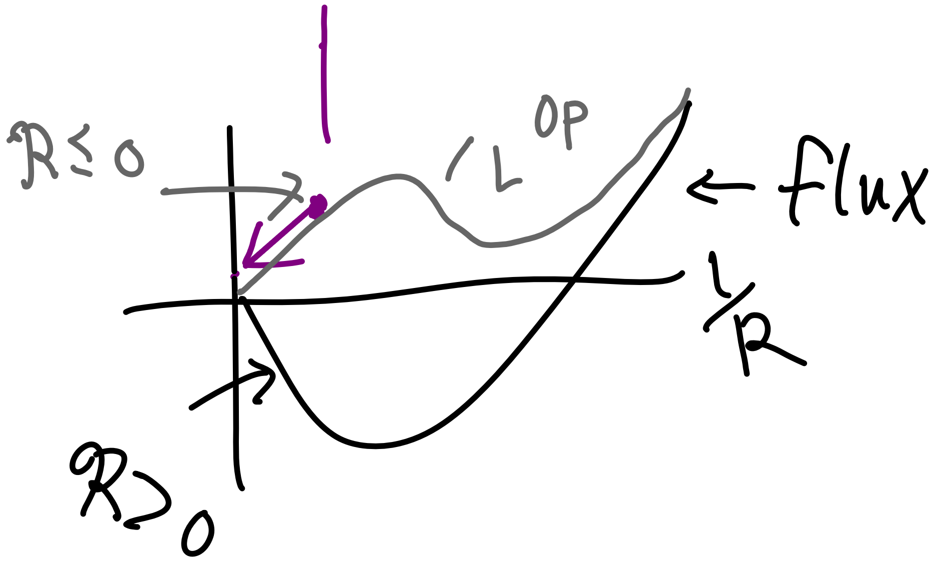


dS_2



dS_3

FRW decay



Again 2-throated warped metric

$$ds^2 = C^2(\tau^{2/C} - w^2)^{C-1}dw^2 + (1 - \frac{w^2}{\tau^{2/C}})^{C-1}(-d\tau^2 + C^2\tau^2 dH_{d-1})$$

Previously found (Dong, Horn, ES, Matsuura, Torroba '12) entropy bound and d.o.f. count grows like τ. GR_d also decouples at late times.

Geodesic => $S_{RT} \sim \tau_1$.

Future/ongoing Directions:

*At least naively, get volume law for smaller entangling regions. cf Balasubramanian/McDermott/Van Raamsdonk, Karch/Uhlemann Mollabashi/Shiba/Takayanagi (integrating out internal d.o.f.) One Hubble patch has ~$c\pi/3$ d.o.f. (`inside the matrix' cf Susskind/Witten '98).

*Entropies in FRW decay

*Calculations of dS_{d+1} correlators generalizing HKLL

*Analyze interaction Hamiltonian and Ψ in simple/toy models (e.g. SYK x SYK with irrelevant couplings, cf Maldacena Qi) and in D1D5 system.