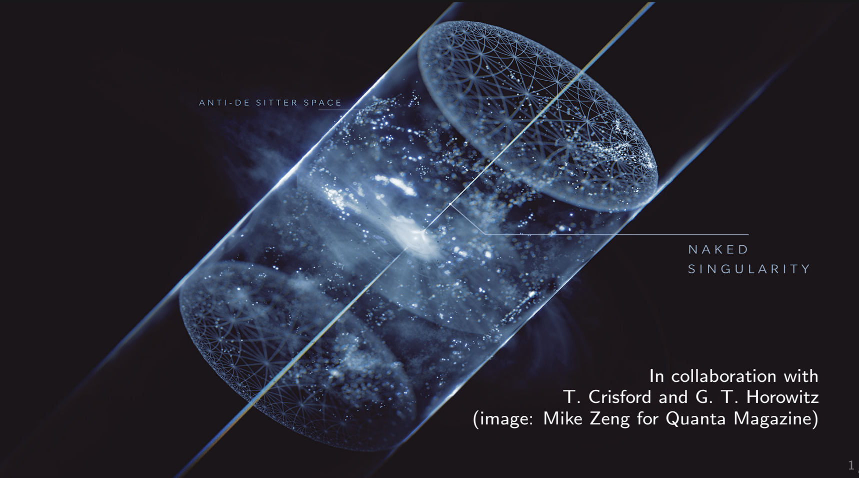
Connecting the weak gravity conjecture to the weak cosmic censorship



In collaboration with
T. Crisford and G. T. Horowitz
(image: Mike Zeng for Quanta Magazine)

Weak Cosmic Censorship - 1/2:

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Weak cosmic censorship (executive summary):

Is it possible to **form a region of arbitrarily large** curvature that is visible to **distant observers**?

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Weak cosmic censorship (Geroch and Horowitz 79):

Let (Σ, h_{ab}, K_{ab}) be a geodesically complete, **asymptotically flat**, initial data set. Let the matter fields obey second order **quasi-linear hyperbolic** equations and satisfy the **dominant energy condition**. Then, generically, the maximal development of this initial data is an asymptotically flat spacetime (in particular $\mathcal{I}^+$ **is complete**) that is strongly asymptotically predictable.

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Claims to fame - Gregory Laflamme type - $d \geq 5$:

Lehner, Pretorius '10 - Black String

Figueras, Kunesch, and Tunyasuvunakool '16 - Black Rings

Figueras, Kunesch, Lehner, and Tunyasuvunakool '17 - Myers-Perry

Asymptotically locally
AdS spacetimes

Anti-de Sitter spacetime - 1/4

Anti-de Sitter space is a maximally symmetric solution to

$$R_{ab} = -\frac{d-1}{L^2} g_{ab}$$

which in global coordinates can be expressed as

$$\mathrm{d}s^2 = -\left(\frac{r^2}{L^2}+1\right)\mathrm{d}\tau^2 + \frac{\mathrm{d}r^2}{\frac{r^2}{L^2}+1} + r^2 \mathrm{d}\Omega^2_{d-2}.$$

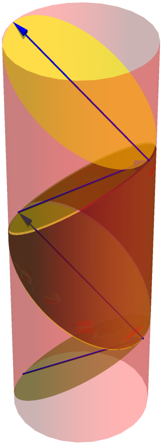
The Poincaré coordinates

$$\mathrm{d}s^2 = \frac{L^2}{z^2}\left[-\mathrm{d}t^2 + \underbrace{\mathrm{d}\mathbf{x}\cdot\mathrm{d}\mathbf{x}}_{(d-2)-\text{coordinates}} + \mathrm{d}z^2\right]$$

do not cover the entire spacetime.

Anti-de Sitter spacetime - 2/4

Conformally, AdS looks like the interior of a cylinder



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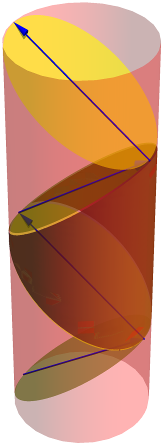

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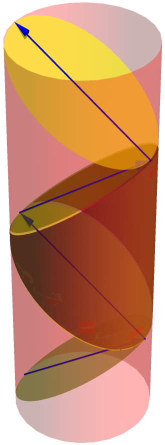

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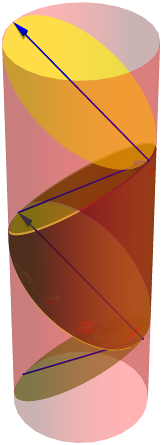

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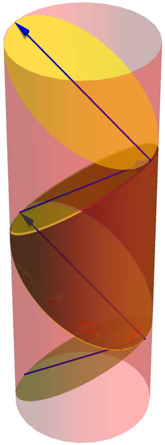

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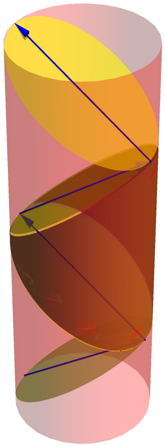

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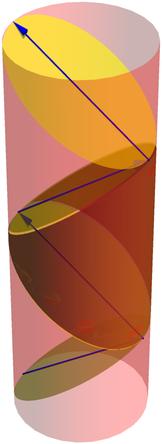

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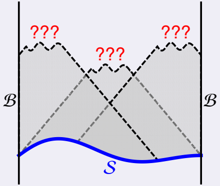
The conformal boundary $\mathcal{B}$ is a **timelike boundary**: need to prescribed **boundary conditions** there.



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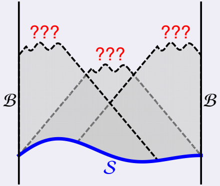

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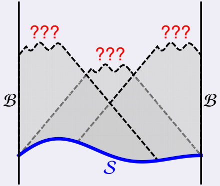

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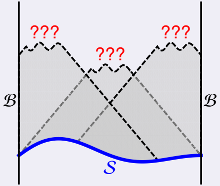

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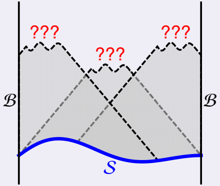

We need to prescribe one **free function of the boundary coordinates** for each **physical degree of freedom**.

Anti-de Sitter spacetime - 4/4

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Wish list:

Remain in **4D**, and start in the **vacuum of the theory**.

Action!

Take the following bulk action:

$$S = \frac{1}{16\pi G}\int \mathrm{d}^4x\,\sqrt{-g}\left[R - F^{ab}F_{ab} + \frac{6}{L^2}\right]\,,$$

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- We **will choose to** dial A_∂ (more precisely $F_\partial = \mathrm{d}A_\partial$).

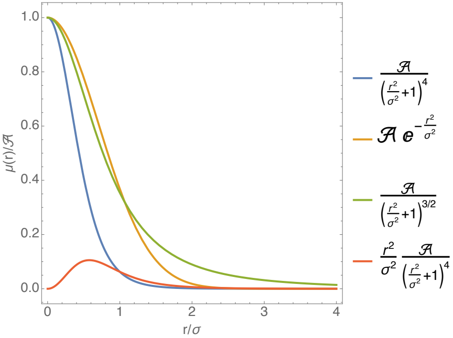
Adiabatic approximation

G. T. Horowitz, N. Iqbal, JES, B. Way '14

M. Blake, A. Donos, D. Tong '14

Adiabatic approximation - 1/2

We have considered many distinct profiles, *e.g.*:



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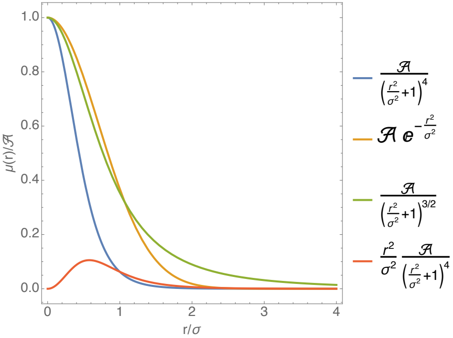

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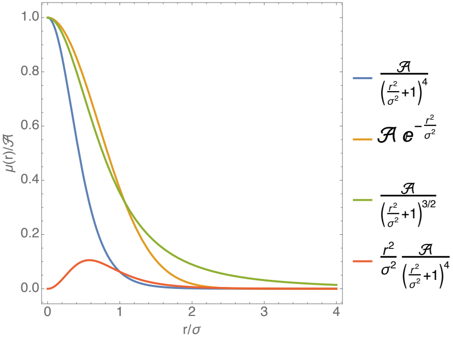

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- In all the cases shown, the IR geometry is always AdS_4.

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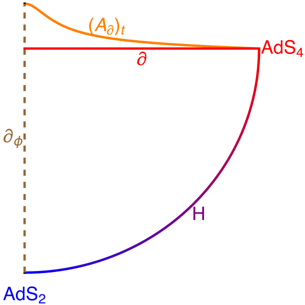

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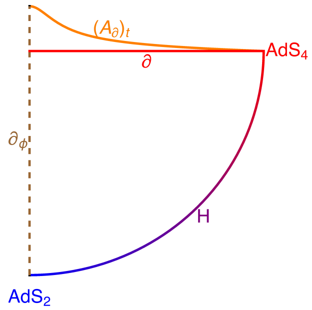

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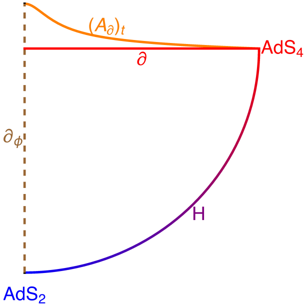
$$a_{\max} = \sqrt{138 + 22\sqrt{33}}/24 \, .$$



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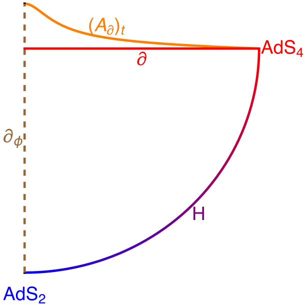

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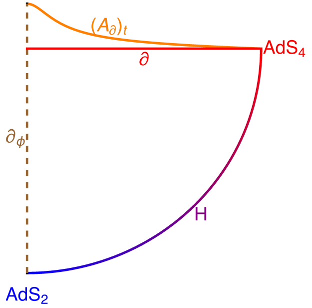
- Smallest $a_{\max}$ amongst all profiles: ought to be **easier for numerics**.



The conjecture

G. T. Horowitz, JES, B. Way '16

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- Impose **boundary electric profile** (with $n \geq 1$):

$$f = \frac{a(t)\, r\, n}{\sigma^2 \left(1 + \frac{r^2}{\sigma^2}\right)^{\frac{n}{2}+1}} \mathrm{d}t \wedge \mathrm{d}r \,.$$

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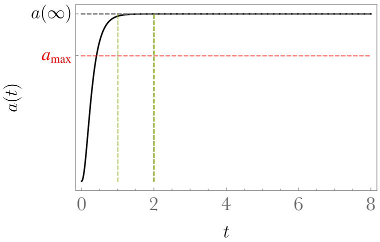

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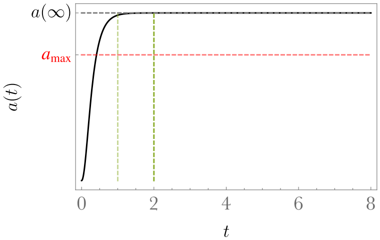

- Smallest $a_{\max} \approx 0.678$ occurs for $n = 1$, so we will use this profile for the full **2+1 time-dependent** case.

The conjecture 2/2

Possible outcomes:

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Conjecture - G. T. Horowitz, JES, B. Way '16:

For $a(\infty) > a_{\max}$, the resulting time evolution leads to **arbitrarily large curvatures** at late times, which are visible to boundary observers: **weak cosmic censorship is violated**.

Violation of the Weak Cosmic Censorship Conjecture

T. Crisford, JES '17

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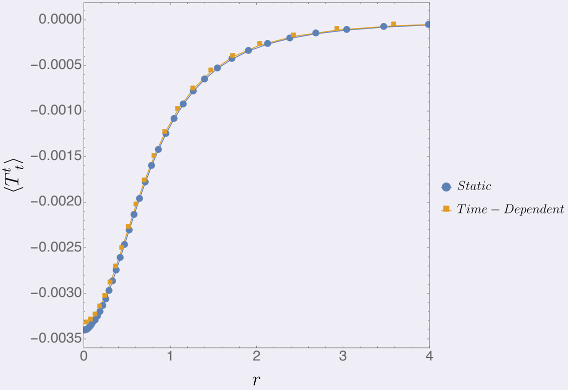
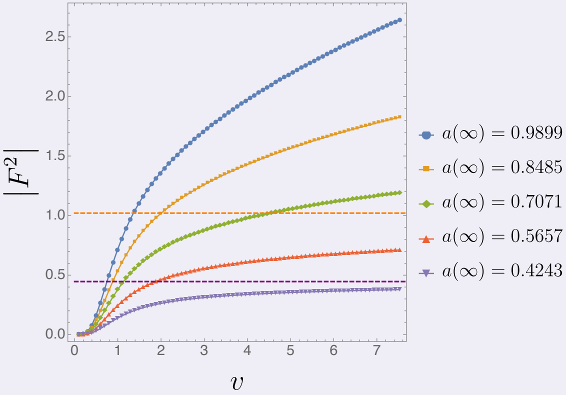


Matching old results with a time-dependent code!

Results 2/2 - $a(\infty) > a_{\max}$:



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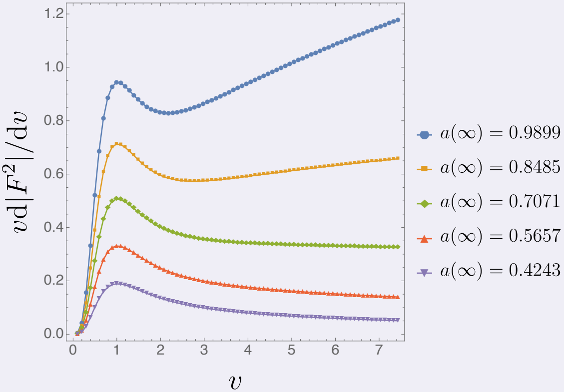

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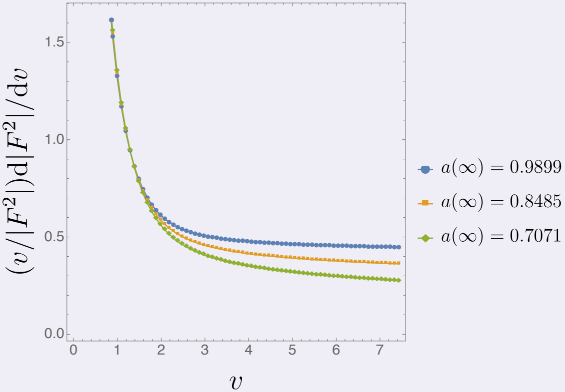

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A dialog

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- **Cumrun**: 'This is very reminiscent of the weak gravity conjecture!'
- **Gary & Jorge & Toby**: 'Let us do this with a charged scalar field in AdS!'

The Weak Gravity Conjecture

N. Arkani-Hamed, L. Motl, A. Nicolis and C. Vafa, '07

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  - Left with 10^{100} Planck scale remnants!

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The weak gravity conjecture in essence ensures that extremal charged black holes are quantum mechanically unstable to Schwinger pair production.

Weak Gravity Conjecture in AdS:

- Schwinger pair production implies **superradiance**:

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where μ is the chemical potential of an **arbitrarily small** RN-black hole. In AdS, becomes a **classical instability**.

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Key Question:

If we take the Weak Gravity Conjecture seriously and include a scalar field with $qL \geq \Delta$ in our action, does our counter-example to Cosmic Censorship still work?

Adiabatic approximation Reloaded

T. Crisford, G. T. Horowitz, JES '17

Take the following bulk action:

$$S = \frac{1}{16\pi G}\int \mathrm{d}^4x\,\sqrt{-g}\left[R - F^{ab}F_{ab} - (\mathcal{D}_a\Phi)(\mathcal{D}^a\Phi)^\dagger - m^2\Phi\Phi^\dagger + \frac{6}{L^2}\right],$$

where $\mathcal{D} = \nabla - i\,q\,A$.

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4. Compute QNMs: if they are **unstable to forming scalar hair**, Cosmic Censorship is likely **preserved**.

Results 1/2:

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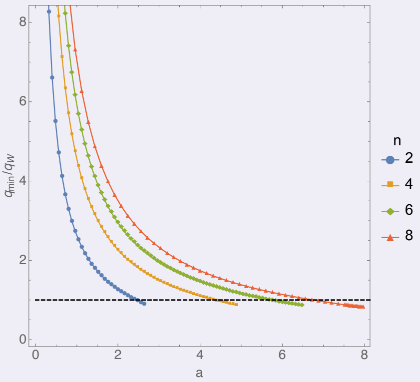

Minimum charge required for a zero-mode to exist with $\Delta = 4$.

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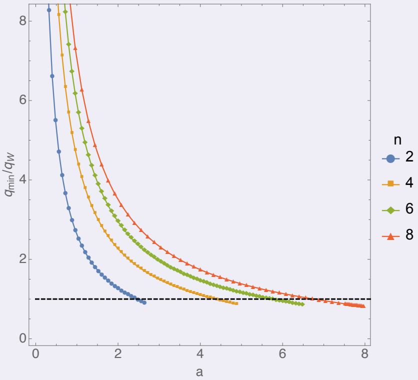

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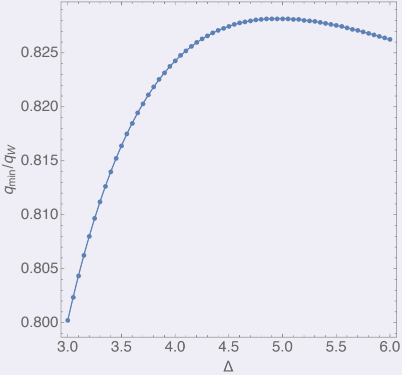

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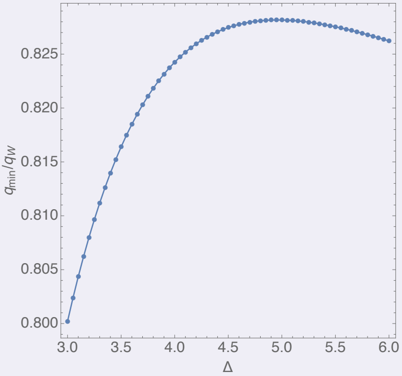
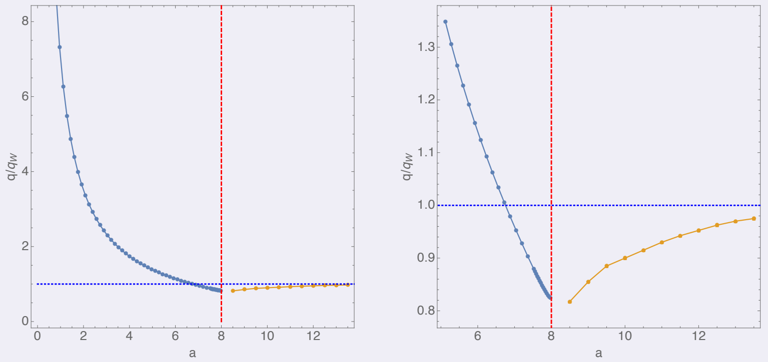


Could the full non-linear solutions with scalar hair also become singular?

Results 2/2:



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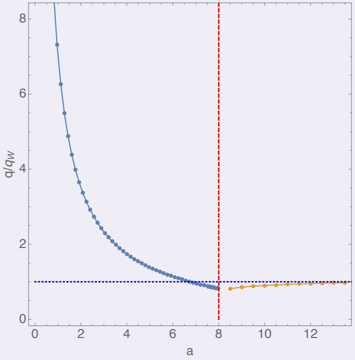

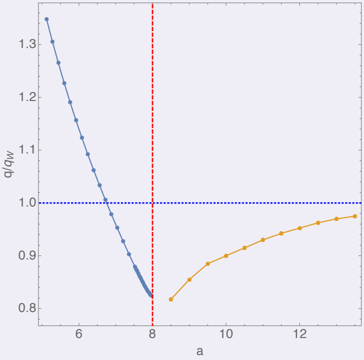

Caveat:

Hovering Black Holes **could** form now that charged matter is present, and would be an **alternative** way of avoiding Cosmic Censorship Violation.

Conclusions:

- We have found a four-dimensional counterexample to the weak cosmic censorship.
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Outlook:

- What is the field theory interpretation of this phenomenon?
- Repeat the time-dependent setup including charge scalars.

Thank You!